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Collective Magnetoplasma Excitations in Two-Dimensional Electron Rings

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The spectra of magnetoplasma excitations in two-dimensional electron disks and rings are studied by optical detection of resonance microwave absorption. For ring-shaped structures, two types of edge magnetoplasma modes localized along the inner and outer boundaries of the ring are observed. It is shown that the interaction between these modes leads to a strong modification of their magnetic-field dependences as compared to disks. In addition to the longitudinal edge magnetoplasma excitations, transverse plasma modes associated with the electron density oscillations along the ring radius are revealed. The spectra of magnetoplasma excitations are calculated in terms of the electrodynamic theory for both ring-shaped and disk-shaped structures. The classification of all modes of collective magnetoplasma excitations observed in the experiment is performed on the basis of the comparison between experimental and theoretical results. © *2004 MAIK "Nauka/Interperiodica".*

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Low-dimensional electron systems in semiconductor structures are the object of numerous experimental and theoretical studies. Specifically, considerable interest has been expressed in studying magnetoplasma excitations in two-dimensional electron systems confined to a certain geometry. Most publications concerned with this subject consider the properties of 2D and 1D magnetoplasma excitations in structures shaped as disks, quantum dots, antidots, and quantum wires [1–6]. Meanwhile, structures with the ring geometry were relatively poorly investigated from both theoretical and experimental points of view. In this geometry, by varying the ratio of the outer a and inner b diameters of the ring, one can study the transition from two-dimensional ($a/b \gg 1$) to one-dimensional ($a/b \sim 1$) plasma excitations. Another attractive feature of the ring geometry is the possibility to study the interaction between two spatially close magnetoplasma edge modes and the dependence of this interaction on distance, electron density, and magnetic field. The few experimental [7] and theoretical [8] attempts to study 2D structures with a ring geometry did not clarify the question about the modification of plasma excitations at the transition from the one-dimensional to two-dimensional case, because these studies included no comparative analysis of rings with different ratios a/b and no comparison between the spectra of plasma excitations in disks and rings. In addition, in the experiment [7], not all magnetoplasma excitation modes characteristic of the ring geometry were observed and the dependence of the energy of these modes on the 2D carrier concentration was not investigated.

The present paper reports on the study of the collective excitation spectrum of a ring-shaped 2D electron system. A comparative analysis of the excitation spectra measured for a ring and a disk with the same outer diameter is performed, and the modification of the magnetoplasma excitation modes depending on the dimensions, magnetic field, and 2D electron concentration is investigated.

The measurements were carried out on two *n*-type GaAs/AlGaAs structures (300-Å-wide single quantum wells) with electron densities of 2.5×10^{11} and 0.8×10^{11} cm^{-2}. On these substrates, mesas were fabricated (by photolithography) in the form of rings with the outer diameter $a = 0.6$ mm and the inner diameter $b = 0.2$ mm. For comparison, disks with a diameter of 0.6 mm coinciding with the outer diameter of the rings were fabricated on the same substrates. The spectra of the dimensional magnetoplasma resonances were measured by the method of optical detection of microwave absorption [9, 10], which provides a high sensitivity. We studied differential (with respect to the microwave power) spectra of recombination radiation of 2D electrons at a temperature $T = 1.5$–4.2 K in the frequency range of microwave excitation from 4 to 50 GHz. The photoexcitation and the reception of the recombination radiation were performed via a quartz optical fiber, which led directly to the sample. The optical signal was

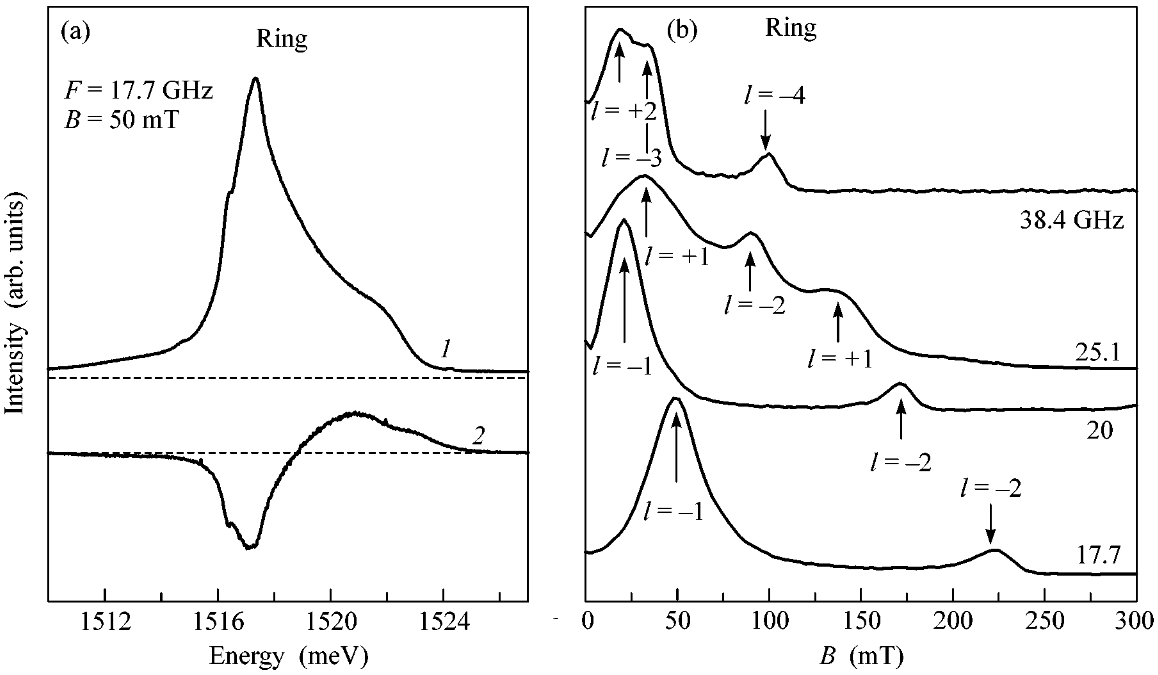

Fig. 1. (a) (*1*) Experimental luminescence spectrum of radiation and (*2*) the differential spectrum with respect to the microwave power for a structure with a ring geometry (a = 0.6 and b = 0.2 mm) in the resonance magnetic field B = 50 mT at a microwave excitation frequency of 17.7 GHz. (b) Typical magnetic-field dependences of resonance absorption measured for the same structure with different microwave radiation frequencies. The 2D electron concentration is 2.6×10^{11} cm^{-2}.

received by a high-sensitivity CCD camera and analyzed by a double spectrometer with a spectral resolution of 0.03 meV. The microwave radiation was supplied to the sample either through the microwave channel or through a coaxial microwave cable providing the high-frequency power transmission in the frequency range of 0–50 GHz with an attenuation smaller than 5 dB. The absolute value of the difference signal was integrated over the whole spectrum of the recombination radiation, and the resulting integral intensity of the differential spectrum served as a measure of the intensity of microwave absorption. Thus measured, the intensity of microwave absorption was studied as a function of magnetic field for different microwave excitation frequencies.

Figure 1a shows a typical radiation recombination spectrum of 2D electrons (the upper curve) and the corresponding differential spectrum with respect to the microwave power (the lower curve); the spectra were obtained at T = 1.5 K and a 2D electron concentration of 2.6×10^{11} cm^{-2} under the resonance conditions (f = 17.7 GHz) in magnetic field B = 50 mT. According to this figure, the resonance microwave absorption leads to a change in the shape of the recombination radiation line because of the heating of the electron system due to the resonance absorption of the microwave power. Figure 1b shows the dependences of the resonance absorption on magnetic field for a ring structure (a = 0.6 mm and b = 0.2 mm) with a 2D electron concentration of 2.6×10^{11} cm^{-2} under different microwave excitation frequencies. The arrows indicate the magnetoplasma modes observed in the experiment and classified according to the theoretical calculation (see below). The dependences of the resonance magnetic field measured in this manner on the microwave radiation frequency are shown in Fig. 2a. For comparison, Fig. 2b presents the magnetic-field dependence of the resonance modes for a structure fabricated in the form of a disk with a diameter equal to the outer diameter of the ring on the same substrate. From Figs. 2a and 2b, one can see that, although the electron concentrations and the outer diameters of the structures are equal, their magnetoplasma excitation spectra are radically different.

For the classification of the resonances observed in the experiment, it is convenient to use two quantum numbers that completely describe all resonance modes for both the ring geometry and the disk. One of these numbers is n = 0, 1, 2, …, which corresponds to the number of nodes that occur in the radial direction for the distribution of the charge density perturbation. The other is the azimuthal number l, which describes the angular distribution of the induced charge density and takes on the values 0, ±1, ±2, ±3, …. Applying this terminology to the magnetic-field dependences shown in Figs. 2a and 2b, we can state that the resonance modes observed in our experiment belong to the series of excitations with n = 0 and different l. Specifically, for the

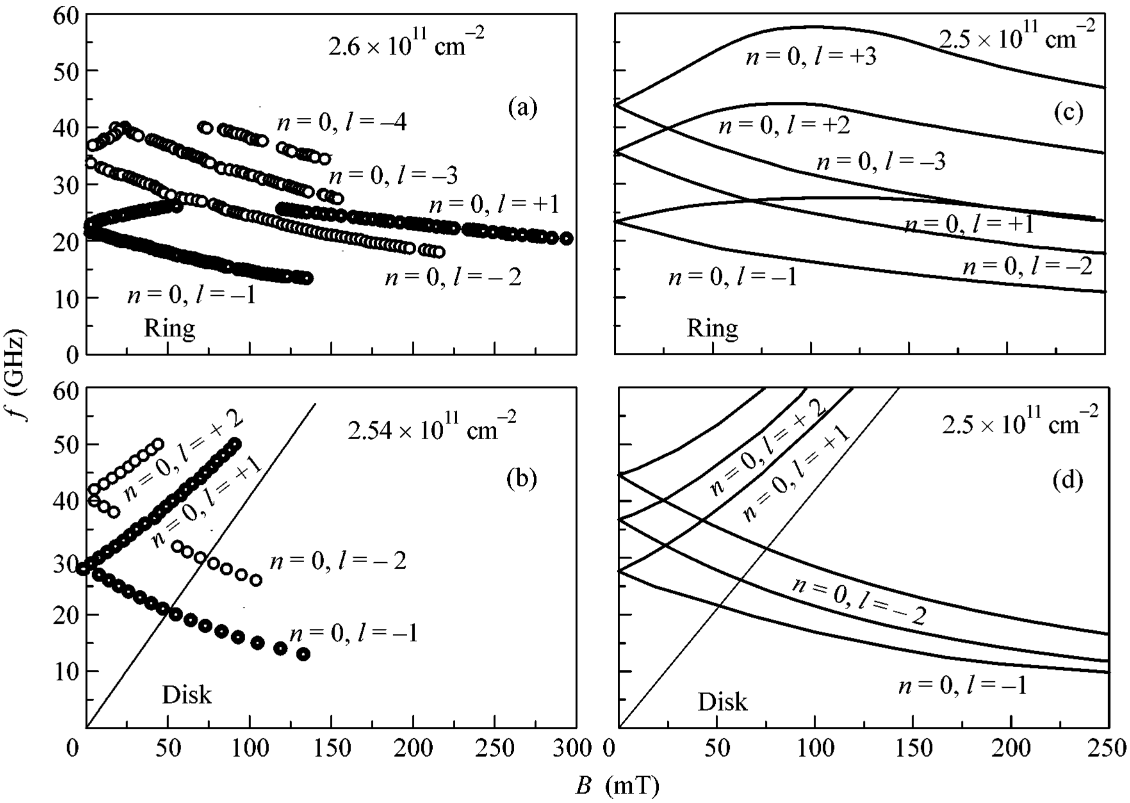

Fig. 2. Experimental magnetic-field dependences of the resonance excitation frequencies of different magnetoplasma modes for (a) a ring structure with $a = 0.6$ mm and $b = 0.2$ mm and (b) a disk with a diameter of 0.6 mm at $n_s = 2.6 \times 10^{11}$ cm^{-2}. Theoretical dependences $f(B)$ for the (c) ring and (b) disk geometries with the same parameters.

ring geometry, modes with $l = \pm1, \pm2, -3$, and -4 manifest themselves, and for the disk geometry, we have modes with $l = \pm1$ and ±2. Despite the identical symmetries and similar choices of quantum numbers, the behavior of magnetoplasma resonances in the case of the ring geometry qualitatively differs from that in the disk. The difference is most pronounced for the upper branch of the lower doublet of the resonance modes (curve $n = 0$, $l = +1$ in Fig. 2a). In the region of low magnetic fields, this mode has a positive magnetodispersion, as well as the corresponding mode in the disk geometry (curve $n = 0$, $l = +1$ in Fig. 2b). However, unlike the case of the disk, where the dependence is monotonic and asymptotically tends to the cyclotron resonance frequency in the region of high magnetic fields, in the case of the ring geometry, it reaches a maximum near 80 mT and then decreases with increasing magnetic field. Simultaneously, a sharp decrease occurs in the oscillator strength of the corresponding resonance. Such a behavior of the resonance mode testifies to its localization near one of the ring edges starting from a certain magnetic field and, hence, points to the edge character of this mode. A similar behavior in high magnetic fields is observed for the lower resonance branch (curve $n = 0$, $l = -1$ in Fig. 2a). This mode initially has a negative magnetodispersion and, as the magnetic field increases, it becomes localized along the outer edge of the ring representing an analogue of the edge magnetoplasma mode with $n = 0$, $l = -1$ in the disk. Proceeding from this fact, we can assume that, in high magnetic fields, the mode with $n = 0$, $l = +1$ is localized and propagates along the inner boundary of the ring, because its excitation energy in these fields considerably exceeds the excitation energy of the lower resonance mode with $n = 0$, $l = -1$ propagating along the outer boundary. In addition to the aforementioned resonances, the magnetoplasma spectrum of the ring contains other edge modes, namely, modes with quantum numbers $l = -2, -3$, and -4. These excitations also propagate along the outer edge of the ring but possess higher energies.

We also carried out the measurements for a ring with the same dimensions but with a lower concentration of 2D carriers: 0.8×10^{11} cm^{-2}. The resulting dependences of the energies of resonance mode excitation on magnetic field are shown in Fig. 3a. As in the case of Fig. 2, Fig. 3b shows the corresponding magnetic-field dependences for a disk with the same concentration of 2D carriers. In addition to the lower resonance modes with $n = 0$, $l = \pm1$, excitations corresponding to $n = 1$ appear in ring-shaped structures with lower charge carrier con-

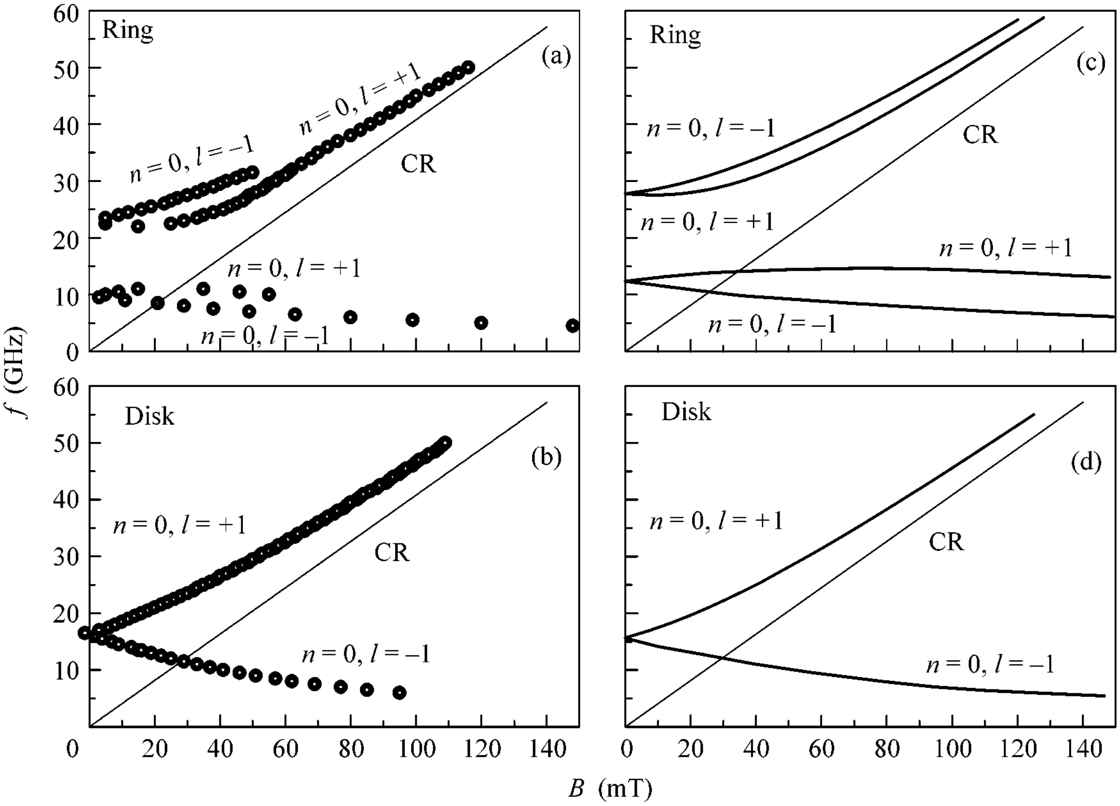

Fig. 3. Experimental magnetic-field dependences of the resonance excitation frequencies of different magnetoplasma modes for (a) a ring structure with $a = 0.6$ mm and $b = 0.2$ mm and (b) a disk with a diameter of 0.6 mm at $n_s = 0.8 \times 10^{11}$ cm^{-2}. Theoretical dependences $f(B)$ calculated for the (c) ring and (d) disk geometries with the same parameters.

centrations. The fundamental difference between these excitations and the resonances with $n = 0$ lies in their behavior in high magnetic fields. The resonance modes with $n = 0$ are localized along the inner and outer edges of the ring, while the upper resonances with $n = 1$ behave in a radically different way. As is seen from Fig. 3a, the lower branch of the resonance doublet with $n = 1$, $l = +1$ has a negative magnetodispersion in low magnetic fields; as the field increases, this branch exhibits a growth and, in high magnetic fields, it tends to the cyclotron resonance frequency, thus displaying the features of a "bulk" magnetoplasmon. By contrast, the upper branch of this doublet with $n = 1$, $l = -1$ initially has a positive magnetodispersion but rapidly decays with increasing magnetic field and, therefore, is only observed in the fields below 50 mT. Note that, unlike the edge modes with $n = 0$, $l = \pm 1$, which are related to the charge density oscillations along the perimeter of the ring, the modes with $n = 1$, $l = \pm 1$ are related to the charge density oscillations along the radius of the ring, and the quantity $\omega_{n=1}(B = 0) = \Omega$ is the corresponding plasma frequency.

In addition to the aforementioned difference between the magnetic-field dependences of resonances in the ring- and disk-shaped structures, the behavior of the magnetoplasma modes exhibit other distinctive features, which testify to the transition from the purely two-dimensional case (disk) to the quasi-one-dimensional case (ring). Primarily, this refers to the lower edge resonance mode in the ring, which exhibits a magnetic-field dependence different from the one characteristic of the disk geometry. Such a behavior can be explained by the strong interaction of the two edge modes corresponding to the outer and inner edges of the ring. This interaction is almost completely absent in the ring with a small inner diameter ($a/b \gg 1$)), and, as a consequence, the plasma frequency at $B = 0$ and the dependence of the resonance frequencies on magnetic field should in this case only slightly differ from those obtained for the disk geometry. Figure 4a illustrates the comparison of the magnetic-field dependences obtained for the frequencies of the lower edge magnetoplasma modes, $n = 0$, $l = -1$, localized at the outer perimeter of the structures in the case of the disk geometry with the diameter $d = 0.6$ mm and in the case of the ring geometry with $a = 0.6$ mm and $b = 0.2$ mm at a 2D electron concentration of 0.8×10^{11} cm^{-2}. As one can see from this figure, in the case of the ring, the plasma frequency at $B = 0$ is almost two times smaller and the field dependence is weaker than in the case of the disk.

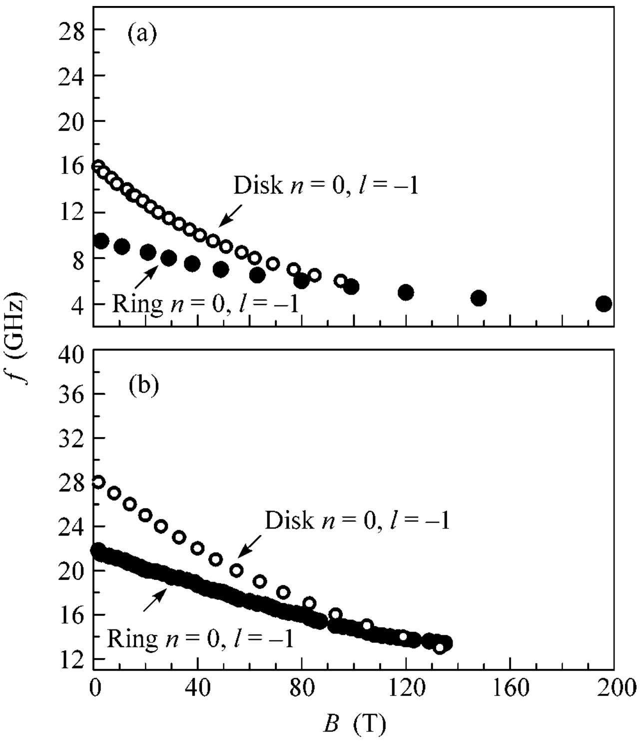

Fig. 4. Comparison of the experimental magnetic-field dependences of the lower resonance mode with $n = 0$, $l = -1$ for a disk with a diameter of 0.6 mm and a ring with a = 0.6 mm and b = 0.2 mm; the 2D electron density concentration is (a) 0.8×10^{11} and (b) 2.6×10^{11} cm^{-2}.

In addition, the characteristic magnetic field at which the resonance frequency decreases by half is greater in the case of the ring: 0.14 T instead of 0.06 T in the case of the disk. Such a dependence of the frequency of the lower resonance mode on magnetic field testifies to a considerable interaction between the inner and outer edge modes. Indeed, according to the theoretical calculations [6], the closer the edge modes, the higher the frequency Ω and the weaker the magnetic-field dependence $\omega(B) \sim \Omega\left(\frac{\Omega^2}{\Omega^2 + \omega_c^2}\right)^{1/2}$. A similar effect of the edge mode interaction is observed for a 2D electron concentration of 2.5×10^{11} cm^{-2} (Fig. 4b).

Our assumptions concerning the classification and character of the magnetoplasma modes observed in the experiment are confirmed by theoretical calculations in terms of the classical electrodynamics [11]. In these calculations, a self-consistent solution is found to the Poisson equation and the continuity equation for the induced density of 2D charge carriers with additional boundary conditions for the radial component of the current. The problem is solved by expanding the concentration in the orthonormal basis of Bessel functions of the first and second kinds, $J_l(\mu_{n,l}r)$ and $Y_l(\mu_{n,l}r)$, in a ring with a sharp potential profile.[1] We used this method to calculate the energies of the eigenmodes of electron density in the ring geometry for different quantum numbers (n, l). Specifically, for modes with $n = 0$ and $l = \pm1$, ±2, and ±3, the corresponding calculated magnetic-field dependences are shown in Fig. 2c. For comparison, Fig. 2d shows the magnetoplasma excitation spectrum calculated for the disk geometry. Figure 3c displays dependences of magnetoplasma resonances on magnetic field that are calculated in a similar way for a ring with a = 0.6 mm and b = 0.2 mm and with an electron concentration of 0.7×10^{11} cm^{-2} and for a disk with a diameter D = 0.6 mm and with a 2D electron concentration of 0.8×10^{11} cm^{-2}. From Figs. 2 and 3, one can see that the experimental and theoretical results are in good agreement, which determines the classification of all magnetoplasma modes observed in the experiment.

Thus, by employing the optical detection method, we experimentally studied the magnetic-field dependences of the resonance excitation frequencies in 2D electron rings. The resonance spectra of rings exhibit two types of modes: the high-frequency "bulk" magnetoplasma excitations, which tend to the cyclotron resonance frequency in high magnetic fields, and a set of edge magnetoplasma modes with lower energies, which propagate along the inner and outer boundaries of the ring. We carried out a classification of the resonances observed in the experiment on the basis of the radial and azimuthal quantum numbers. For comparison with the ring geometry, we used the same method to measure the excitation spectra for a disk with a diameter equal to the outer diameter of the ring. We have shown that a change in the geometry of the structure under study leads to a qualitatively different resonance excitation spectrum. In terms of the electrodynamic theory, we calculated the magnetic-field dependences of resonance excitations for both the ring and the disk. The resulting energies of plasma and magnetoplasma excitations coincide with the resonances observed in the experiment, which testifies to the applicability of the given approximation. All this allows us to construct a full picture of collective excitations of the electron gas in a ring and opens up new possibilities for further studies of 2D edge plasmons. In this respect, the phenomenon of special interest is the transition from 2D to 1D magnetoplasma excitations, which occurs in the ring geometry when the ratio of the outer diameter to the inner one decreases.

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