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# Universal Relation between Hall Conductivity and the Damping Constant of Edge Magnetoplasma Resonances

**M. N. Khannanov[a], V. A. Koval'skiĭ[a], I. V. Kukushkin[a], S. I. Gubarev[a], J. H. Smet[b], and K. von Klitzing[b]**

[a] *Institute of Solid State Physics, Russian Academy of Sciences, Chernogolovka, Moscow region, 142432 Russia*
*e-mail: koval@issp.ac.ru*
[b] *Max-Planck-Institut für Festkörperforschung, 70569 Stuttgart, Germany*

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The spectra of edge magnetoplasma excitations in two-dimensional (2D) electron disks have been analyzed by the method of optical detection of resonant microwave absorption. The magnetic dispersion of an edge magnetoplasmon in samples with a high 2D electron density is found to be poorly reproduced by existing theoretical models. Analysis of the magnetic-field dependence of the linewidth of resonant microwave absorption for samples with various 2D electron densities shows that the inverse width of the main mode of resonant microwave absorption is universally proportional to the Hall resistance of 2D electrons.

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For the last three decades, considerable attention has been focused on the physics of low-dimensional electron systems in certain geometry such as disks, rings, and strips. This interest is primarily stimulated due to the discovery of a new type of collective excitations of the charge density, edge magnetoplasmons that propagate along the edge of such a system and are localized near this edge in strong magnetic fields. The characteristic features of the edge magnetoplasmons are a decrease in the propagation velocity of these waves along the edge of the system with increasing the magnetic field and weak damping of these modes, which decreases with increasing the magnetic field. These properties of the edge magnetoplasmons were examined in numerous experimental [1–4] and theoretical works [5–7]. Most measurements of the widths of magnetoplasma resonances [8–10] were carried out with semiconductor heterostructures in the presence of the quantum Hall effect with the use of the reentrant-cavity method [11]. The magnetic dispersion (the magnetic field dependence of the resonant frequency) and damping of the edge magnetoplasmons in strong fields were also studied in [12] for a single 2D electron disk, where the passage of picosecond pulses through a sample was detected. The damping of a wave package traveling along the edge of the sample was determined in terms of the change in the amplitude of the passed signal. However, this was not a direct measurement of the damping of the main edge magnetoplasmon mode, because harmonics higher in energy could be present in the wave package. According to [10, 12], the damping of the edge magnetoplasmons in the quantum Hall effect mode is determined by the diagonal conductance $\sigma_{xx}$, which contradicts the theoretical results obtained in the limit of the sharp edge of the semi-infinite 2D system [6]. In contrast to [10, 12], Volkov and Mikhaĭlov [6] predicted the linear dependence of the linewidth of the resonant absorption of the edge magnetoplasmons on the Hall conductivity $\sigma_{xy}$ in the low-frequency limit $\omega\tau^* \ll 1$ ($\tau^*$ is the electron elastic relaxation time). To date, no attempts have been made to measure the damping of plasmons in the high-frequency limit ($\omega\tau^* \gg 1$) and to examine how the magnetic dispersion and damping of the plasma mode change as the frequency decreases.

The aim of this work is to comprehensively study the magnetic dispersion of edge magnetoplasma excitations and the damping of these modes as the magnetic field varies from zero to the quantum Hall effect regime in a wide range of the frequency of microwave excitations. This study makes it possible to analyze the transition from the high-frequency limit ($\omega\tau^* \gg 1$) to the low-frequency one ($\omega\tau^* \ll 1$).

Investigations were conducted with *n*-type GaAs/AlGaAs quantum wells with an electron density of $0.7 \times 10^{11}$ to $6.6 \times 10^{11}$ cm$^{-2}$. The mobility $\mu$ of various samples is $(2–8) \times 10^6$ cm$^2$/V s. Mesas in the form of a disk 1 mm in diameter are made in all the structures by means of photolithography. To measure the spectra of the dimension magnetoplasma resonance, the high-sensitive method of optical detection of microwave

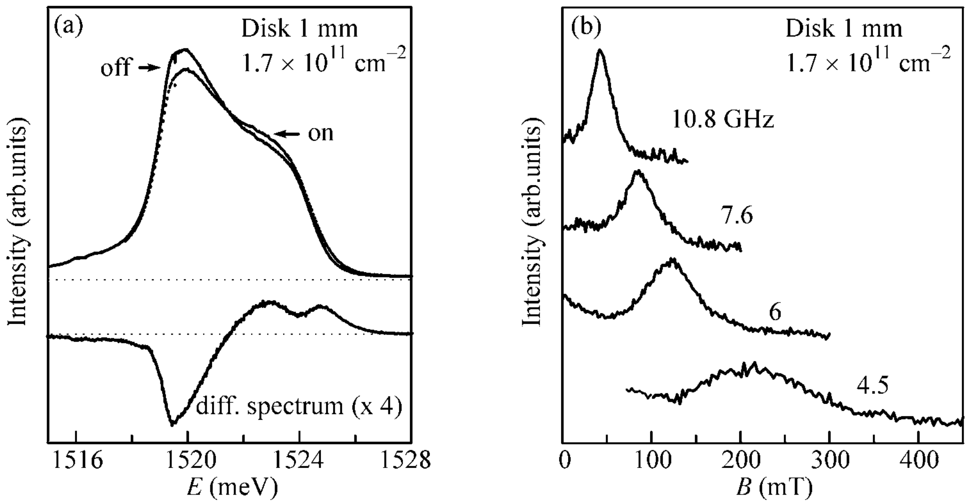


**Fig. 1.** (a, upper part) Luminescence spectra and (a, lower part) the microwave-power-differential radiation spectrum of 2D electrons as measured in a disk 1 mm in diameter in the resonant magnetic field $B$ = 130 mT under 6-GHz microwave excitation. The dotted line is of change in the luminescence spectrum under the resonant microwave power absorption. The density of 2D electrons is equal to $1.7 \times 10^{11}$ cm$^{-2}$. (b) Typical magnetic field dependences of the resonant absorption measured in the same structure for various frequencies of microwave radiation.

absorption is used [13, 14]. In the experiment, we compare the spectra of the recombination radiation of 2D electrons that are measured under microwave irradiation and without it at temperatures of 1.5–4.2 K for microwave excitation frequencies from 0.6 to 50 GHz. For photoexcitation, a 750-nm stabilized semiconductor laser with a power of about 0.1 mW is used. Light from the laser is transferred to the sample through an optical quartz light guide placed so as to ensure the uniform illumination of the surface of the structure under investigation. By means of the same light guide, the photoluminescence signal of 2D electrons is collected; then, this signal is detected by a highly sensitive CCD detector and is analyzed by means of a double spectrometer with a spectral resolution of 0.03 meV. Microwave radiation is transferred from the generator to the sample through either a microwave duct or a coaxial microwave cable, which ends with a radiating dipole antenna.

The spectra of the radiative recombination of the 2D electrons measured in a disk 1 mm in diameter with a 2D-electron density of $1.7 \times 10^{11}$ cm$^{-2}$ in the (solid line) absence and (dotted line) presence of microwave radiation are shown in the upper part of Fig. 1a. The difference between these two luminescence spectra (spectrum differential in the microwave power) is shown in the lower part of Fig. 1a. The measurements were conducted at $T = 4.2$ K under the resonance absorption conditions (the frequency $f = 6$ GHz and the magnetic field $B$ = 130 mT). Owing to the absorption of microwave power, the electron system is heated and, therefore, the shape of the recombination radiation line is changed. The absolute value of the difference signal is integrated over the entire recombination-radiation spectrum, and the integral intensity of the differential spectrum thus determined serves as a measure of the microwave absorption intensity. The magnetic-field dependence of this quantity is analyzed for various microwave excitation frequencies. The typical spectra of resonant absorption for the same sample that are obtained as a result of varying the magnetic field for a fixed microwave frequency are shown in Fig. 1b. The resonances observed in the absorption spectra refer to the main lower mode of magnetoplasma excitations, which has negative magnetic dispersion and exhibits edge behavior in strong fields. Knowing the resonant magnetic field thus measured as a function of the microwave radiation frequency, one can determine the magnetic dispersion of plasma modes and examine the damping of these modes that is determined using the resonant-absorption linewidth.

Figure 2 shows the resonant frequency measured as a function of the magnetic field in disks with various sizes and densities of 2D electrons. In addition to the upper resonance branch corresponding to a bulk magnetoplasmon whose frequency asymptotically approaches the cyclotron frequency in the infinite field limit, the lower branch of the edge magnetoplasmon whose frequency decreases with increasing the magnetic field is observed in the disk. The resonant frequencies of these modes depend both on the 2D electron density and on the disk diameter, which is a result of the mixing of the plasma and cyclotron modes due to the limited sizes of the samples. For the mesa of the disk shape with diameter $d$, the frequencies of the upper and

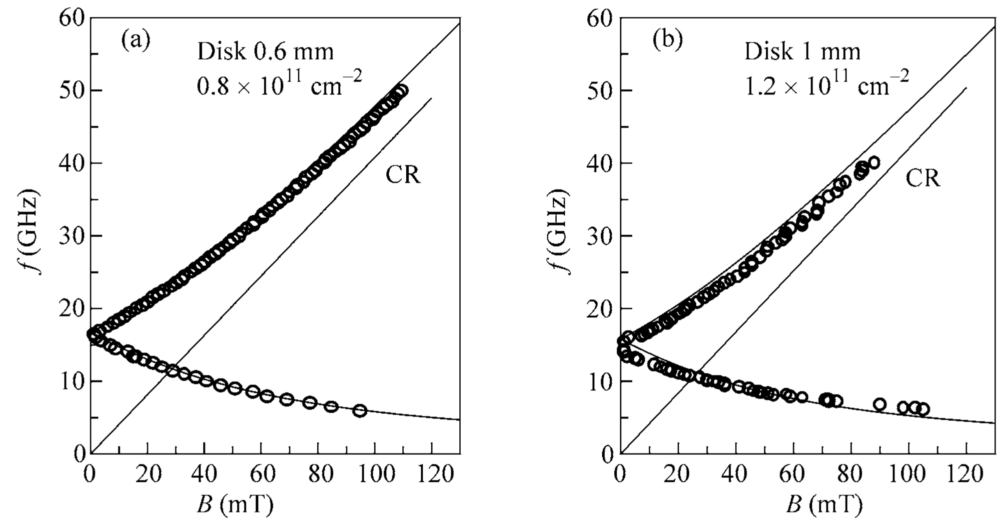


**Fig. 2.** Magnetic-field dependences of the resonant plasma frequencies measured in disks with the diameter $d$ = (a) 0.6 and (b) 1 mm and the 2D electron density $n_s$ = (a) $0.8 \times 10^{11}$ and (b) $1.2 \times 10^{11}$ cm$^{-2}$. The solid line is the theoretical dependence given by Eq. (1).

lower magnetoplasmon branches are described by the expression [1]

$$\omega_{\pm} = \pm\frac{\omega_{CR}}{2} + \sqrt{\omega_p^2 + \left(\frac{\omega_{CR}}{2}\right)^2}. \qquad (1)$$

Here, $\omega_{CR} = eB/m^*$ is the cyclotron frequency and $\omega_p$ is the plasma-oscillation frequency in the disk in the absence of the magnetic field. In the absence of the decay effects, the latter frequency is expressed as [13]

$$\omega_p^2 = \frac{2\pi n_s e^2}{m^* \epsilon_{\text{eff}}} q, \qquad (2)$$

where $\epsilon_{\text{eff}}$ is the mean value of the permittivities of free space and GaAs, $m^*$ is the effective mass of the electron, and $q = 2.4/d$ is the wavenumber of the edge magnetoplasmon [13]. The above dependences well describe the behavior of the magnetoplasmon modes in weak fields in the disk 0.6 mm in diameter with a density of $0.8 \times 10^{11}$ cm$^{-2}$ (Fig. 2a). As the 2D electron density and disk size increase, delay effects associated with the fact that the plasmon velocity becomes comparable with the speed of light become significant [13]. For the disk with an electron density of $1.2 \times 10^{11}$ cm$^{-2}$ and a diameter of 1 mm (see Fig. 2b), owing to delay, plasma modes in the absence of the field begin below the plasma frequency given by Eq. (2) and the slope $d\omega/d\omega_c$ in the limit $B \longrightarrow 0$ is smaller than the standard value of 1/2 obtained from Eq. (1). As the magnetic field increases and the frequency and velocity of the lower mode decrease, the delay effects become weaker and, as a result, the theoretical curve determined by Eq. (1) is expected to well describe experimental points in strong fields. Figure 3 shows the magnetic field dependences of the resonant modes of the edge magnetoplasmons as measured for the disks 1 mm in diameter and the electron densities $1.45 \times 10^{11}$ and $1.85 \times 10^{11}$ cm$^{-2}$ in the magnetic field up to 0.8 T. As seen in this figure, the theoretical curve given by Eq. (1) is below the corresponding experimental curve and intersects the latter at 0.6 T (see Fig. 3a). This discrepancy between the experiment and theory can be attributed to the fact that Eq. (1) is obtained in the oblate-ellipsoid model [1, 15], whereas the equilibrium-density distribution in the disks under consideration is uniform up to the edges and then decreases rapidly in the depletion layer with the characteristic size of several micrometers. The dotted lines in Fig. 3 are the theoretical dependences of the edge magnetoplasmons that are calculated in the model of the sharp edge of the semi-infinite 2D system [6] in the low-frequency limit $\omega\tau^* \ll 1$ {see Eq. (47) in [6]}. Owing to the presence of the magnetic field-dependent logarithmic term in the theoretical formula [6], the decrease in these curves with increasing the field is slower than Eq. (1) and the experimental data, as seen in Fig. 3. Moreover, these theoretical curves lie above the experimental points in contrast to the results obtained by Eq. (1). Such discrepancy between the experiment and theory [1, 6] requires the further examination of the frequency dependence of the edge magnetoplasmons in strong fields in the framework of other theoretical models.

The background for measuring the width of the resonance lines of the edge magnetoplasmons in disks is work [16], where the damping of a 1D plasmon in single electron strips was examined and it was found that the width $\gamma$ of the resonance-absorption line is inversely proportional to the magnetic field. The procedure used in this work does not provide for varying the frequency of the microwave radiation at a fixed magnetic field and directly measuring the $\gamma(B)$ dependence. Instead, the resonance linewidth in the magnetic field units $\gamma_B$ is

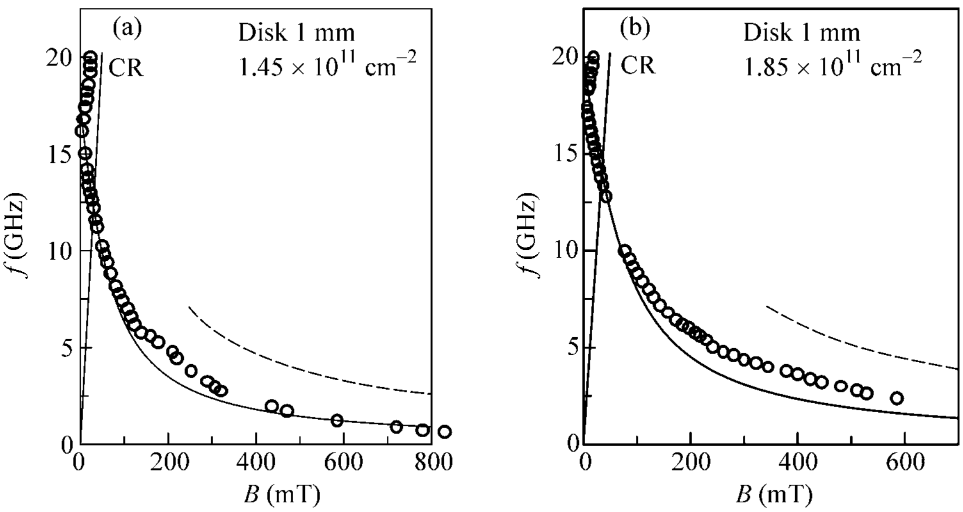


**Fig. 3.** Magnetic-field dependences of the resonant plasma frequencies measured in disks with the diameter $d = 1$ mm and 2D electron density $n_s$ = (a) $1.45 \times 10^{11}$ and (b) $1.85 \times 10^{11}$ cm$^{-2}$. The solid line is the theoretical dependence given by Eq. (1). The dashed line is the theoretical dependence calculated in the low-frequency limit $\omega\tau^* \ll 1$ in [6].

determined from the absorption spectra (see Fig. 1b) and this value is multiplied by the absolute value of the magnetic dispersion slope $|df/dB|$ in the corresponding magnetic field calculated from the experimental curves shown in Figs. 2 and 3. Figure 4 shows the quantity $1/\gamma$ measured as a function of the Hall resistance $\rho_{xy} = B/enc$ for the disks 1 mm in diameter with various densities and mobilities of 2D electrons. The density and mobility of electrons in the 2D structures vary in the ranges (1.4–6.6) × $10^{11}$ cm$^{-2}$ and (2–8) × $10^6$ cm$^2$/V s, respectively. The width of the edge magnetoplasmon line in weak magnetic fields in the limit $B \longrightarrow 0$ varies widely in various samples, depending on the mobility of the electrons and spacer thickness. An absolutely different situation is observed in the limit of strong magnetic fields, where, despite the difference in the $\gamma(0)$ values, the widths of the resonance lines of the edge magnetoplasmons in all the structures approach one universal value determined by the Hall conductivity $\sigma_{xy}$. Such a result is predicted by the theory developed in [6] according to which the damping of the edge magnetoplasmon in the limit $\omega\tau^* \ll 1$ is described by the expression

$$\gamma = \pi q \sigma_{xy}/\epsilon_{\text{eff}}. \quad (3)$$

Here, $\epsilon_{\text{eff}}$ is the average value of the permittivities of free space and GaAs and $q = 2.4/d$ is the wavenumber of the edge magnetoplasmon [13]. As seen in Fig. 4, the theoretical dependence [3] shown by the solid line is in satisfactory agreement with the experimental data for the inverse width of the absorption line of the edge magnetoplasmons for various structures in strong fields. This result contradicts the experimental data obtained in [10, 12], according to which the damping of the edge magnetoplasmons is determined by the diagonal conductivity $\sigma_{xx}$. We note that the universal relation found in this work between the width of the resonant microwave absorption line of the edge magnetoplasmons and the Hall conductivity is observed up to sufficiently strong magnetic fields $B = 1$ T; for this reason, the quantization regime for the Hall resistance can be achieved at $T = 1$–$2$ K. Thus, manifestation of the quantization of the Hall conductivity can be studied through the quantization of the width of the edge magnetoplasma resonances. Such investigations can reveal the cause of the quantization of the Hall resistance and separate the bulk and edge contributions to the quantum

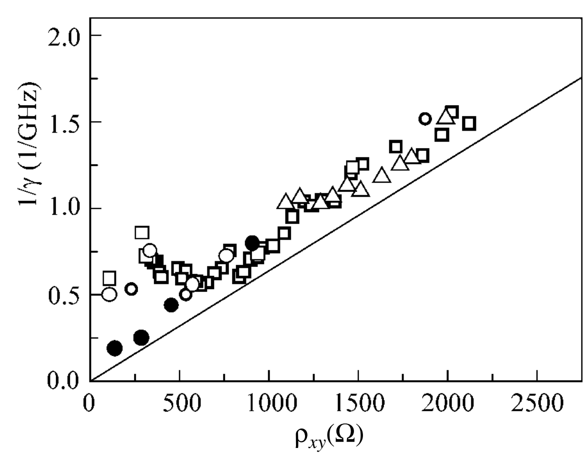


**Fig. 4.** Inverse width $1/\gamma$ of the lower magnetoplasma mode vs. $\rho_{xy} = B/enc$. The points are experimental data obtained for samples with various mobilities and densities. The density and mobility of electrons in these structures vary in the ranges (1.4–6.6) × $10^{11}$ cm$^{-2}$ and (2–8) × $10^6$ cm$^2$/V s, respectively. The solid line is the theoretical dependence given by Eq. (3).

Hall effect. Moreover, since incompressible quantum strips whose number is determined by the filling factor appear near the edge of the sample in the quantum Hall effect regime [17], we hope to study manifestation of these incompressible strips in the edge magnetoplasmon spectra in the quantum Hall effect regime. The prospect of analyzing the temperature dependence of the frequency of the edge magnetoplasmons under the quantum Hall effect conditions is also interesting in view of the possibility of measuring the energy gap $\delta$ from the temperature dependence. Indeed, according to [6], the frequency of the edge magnetoplasmons in the low-frequency limit $\omega\tau^* \ll 1$ in the quantum Hall effect regime is inverse proportional to the temperature and the corresponding coefficient depends both on the quantized values of the Hall conductivity and on the energy gap $\delta$:

$$\omega(q) \sim \frac{2q\sigma_{xy}}{\epsilon_{\rm eff}} \ln\frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{\delta}{T}, \tag{4}$$

where $\sigma_{xx}$ depends exponentially on the electron temperature $T$ in the quantum Hall effect regime. The $\delta$ value can be determined from this expression if the wavenumber $q$ and resonant frequency of the edge magnetoplasmons are reliably measured.

Thus, the edge magnetoplasmon modes have been examined in this work in the resonance absorption spectra measured for disks with various mobilities and densities of 2D electrons. The behavior of an edge mode in the magnetic field in the transition from the high-frequency limit to the low-frequency one has been qualitatively analyzed. The damping of the edge magnetoplasmons for all the samples in strong fields has been found to depend only on the off-diagonal component of the conductivity as $\gamma \sim \sigma_{xy}$. The theory developed in [6] has been shown to well describe the damping of magnetoplasmons in the fields up to 1 T.

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