Introduction to topological aspects in condensed matter physics

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2nd lecture

1. Topological insulators w/ TRS in 2D & 3D
   - Z_2 invariant for 2D & 3D topological insulators
   - Experimental detection of 2D& 3D topological insulators

2. Topological superconductors in 1D
   - BdG theory for superconductors
   - Topological superconductors in 1D: Kitaev model
   - Majorana edge states
   - InSb nanowire-heterostructure

3. Topological superconductors in 2D (w/o TRS)
   - Topological superconductors in 2D: chiral p-wave SC
   - Majorana edge and vortex-bound state
   - Sr_2RuO_4

Band theory of solids and topology

crystal momentum

Bloch Hamiltonian $H(\mathbf{k})$

Bloch wavefunction

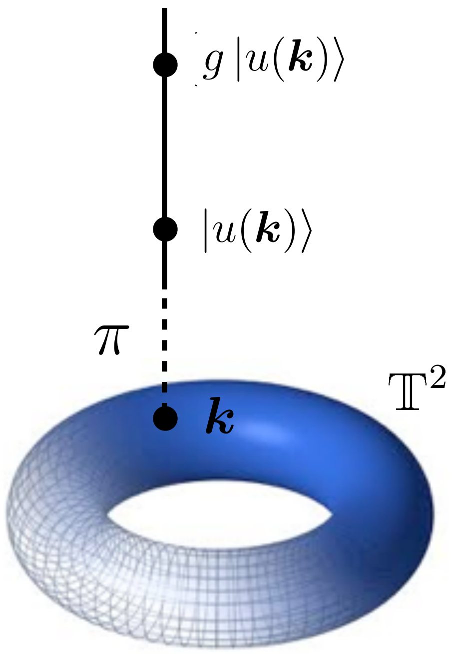
$$H(\boldsymbol{k})\,|u_n(\boldsymbol{k})\rangle = E_n(\boldsymbol{k})\,|u_n(\boldsymbol{k})\rangle$$

Symmetry protected topological phases:

Study topology of the following mapping:

$$\text{Brillouin zone} \longmapsto H(\boldsymbol{k})$$

Hamiltonians
with energy gap
satisfying certain symmetry operations

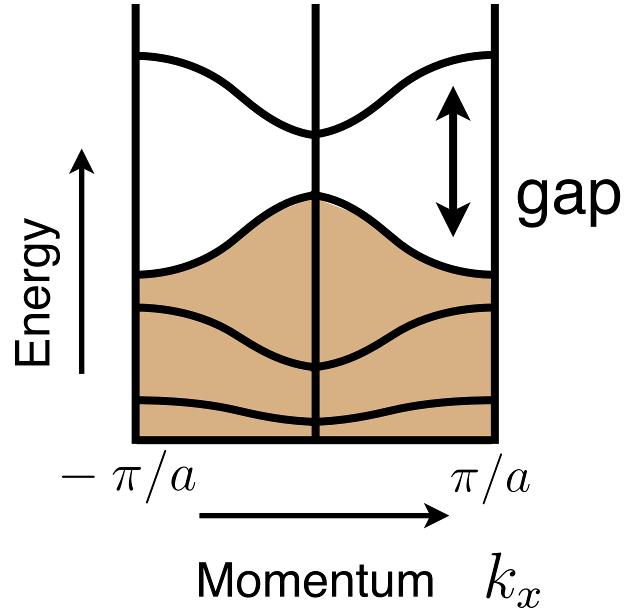


Topological equivalence:

Band structures are equivalent if they can be continuously deformed into one another without closing the energy gap and **without breaking the symmetries of the system**.

Symmetries to consider:

- time-reversal symmetry (anti-unitary)
- particle-hole symmetry (anti-unitary)
- sublattice symmetry and other crystallographic symmetries



Note: *Unitary* symmetries $[H(\mathbf{k}), U_{\mathrm{S}}] = 0$ (e.g. spin-rotation symmetry) can be removed by block-diagonalizing $H(\mathbf{k})$

Time-reversal symmetry & Kramers theorem

Presence of time-reversal symmetry gives rise to new topological invariants **[Kane-Mele, PRL 05]**

$$\Theta : \quad t \to -t, \quad \mathbf{k} \to -\mathbf{k}, \quad \hat{S}^\mu \to -\hat{S}^\mu$$

Time-reversal symmetry implemented by anti-unitary operator:

$$\Theta = U_{\mathrm{T}} \mathcal{K} = e^{i\pi \hat{S}^y/\hbar} \mathcal{K}$$

complex conjugation operator

$$\Theta \psi = e^{i\pi \hat{S}^y/\hbar} \psi^*$$

For quadratic Hamiltonians in momentum space: $\Theta \mathcal{H}(\mathbf{k}) \Theta^{-1} = +\mathcal{H}(-\mathbf{k})$

For spin-$\frac{1}{2}$ particles: $\Theta^2 = -1 \qquad U_{\mathrm{T}} = -U_{\mathrm{T}}^T \qquad \Theta = i\sigma_y \mathcal{K} \qquad \Theta \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = \begin{pmatrix} \psi_\downarrow^* \\ -\psi_\uparrow^* \end{pmatrix}$

Kramers theorem (for spin-1/2 particles): $\Theta^2 = -1 \quad \Rightarrow \langle \psi | \Theta \psi \rangle = -\langle \psi | \Theta \psi \rangle = 0$

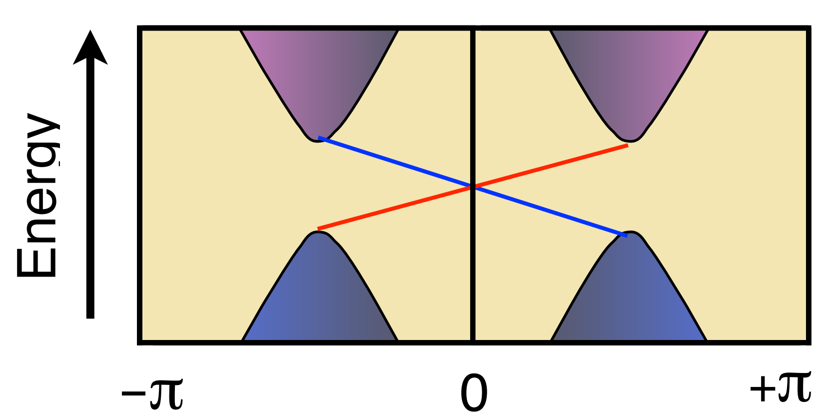
$\Rightarrow$ all eigenstates are at least two-fold degenerate

$\Rightarrow$ for Bloch functions in k-space:

$|u(\mathbf{k})\rangle$ and $|u(-\mathbf{k})\rangle$ have same energy; degeneracy at TRI momenta

Consequences for edge states:

— states at time-reversal invariant momenta are degenerate
— crossing of edge states is protected
— absence of backscattering from non-magnetic impurities



Time-reversal-invariant topological insulator

2D topological insulator: [Bernevig, Hughes, Zhang 2006]

(also known as Quantum Spin Hall insulator) [Kane-Mele, PRL 05]

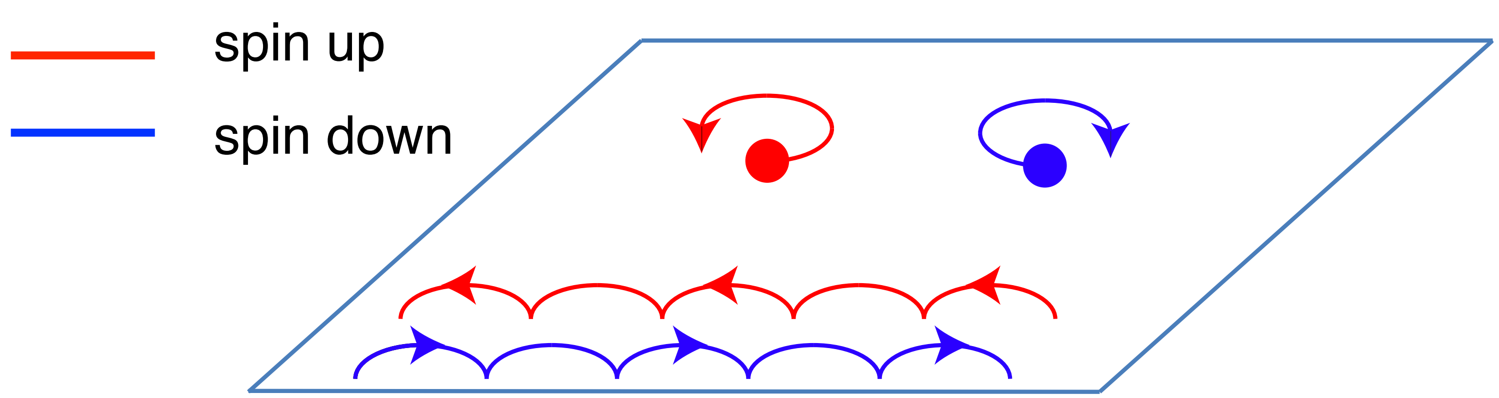
2D Bloch Hamiltonians in the presence of time-reversal symmetry:

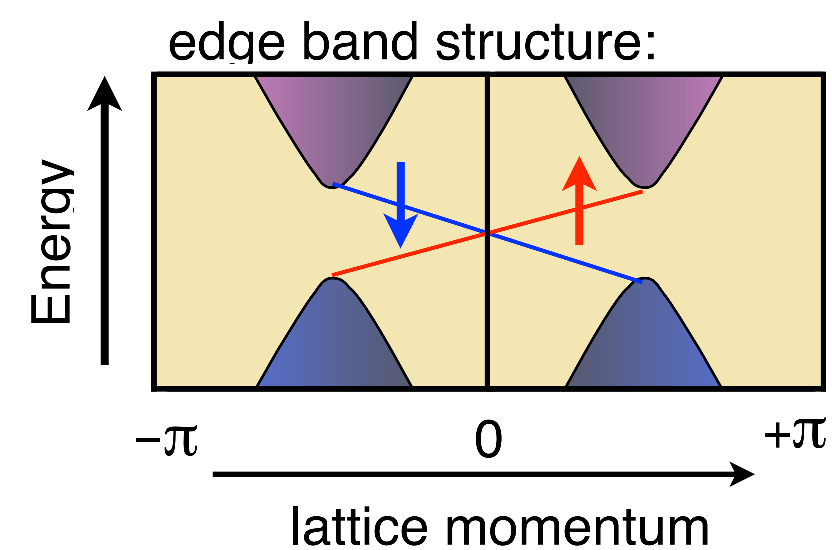
$$\boxed{\Theta\mathcal{H}(\mathbf{k})\Theta^{-1} = +\mathcal{H}(-\mathbf{k})} \qquad \Theta = i\sigma_y \otimes \mathbb{1}\,\mathcal{K} \qquad \Theta^2 = -1$$

Simplest model:
(Chern insulator)^2

$$\mathcal{H}(k_x, k_y) = \begin{pmatrix} H_\uparrow & 0 \\ 0 & H_\downarrow \end{pmatrix} = \begin{pmatrix} H_{\mathrm{CI}}(\mathbf{k}) & 0 \\ 0 & H^*_{\mathrm{CI}}(-\mathbf{k}) \end{pmatrix}$$

S^z is conserved





Bulk energy gap but gapless edge: Spin filtered edge states

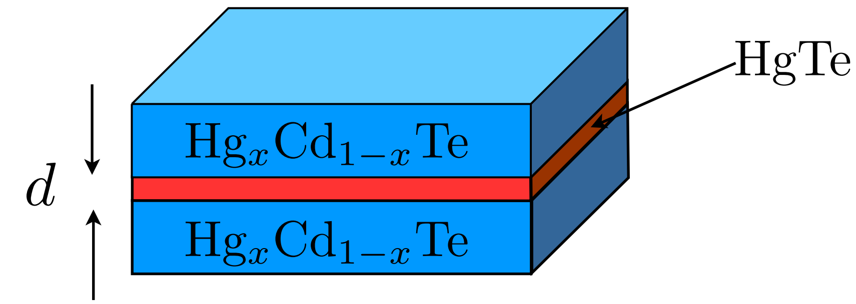
— protected by time-reversal symmetry
— half an ordinary 1D electron gas
— is realized in certain band insulators with strong spin-orbit coupling

TRI topological insulator: HgTe quantum wells

- observed in HgTe/(Hg,Cd) quantum wells

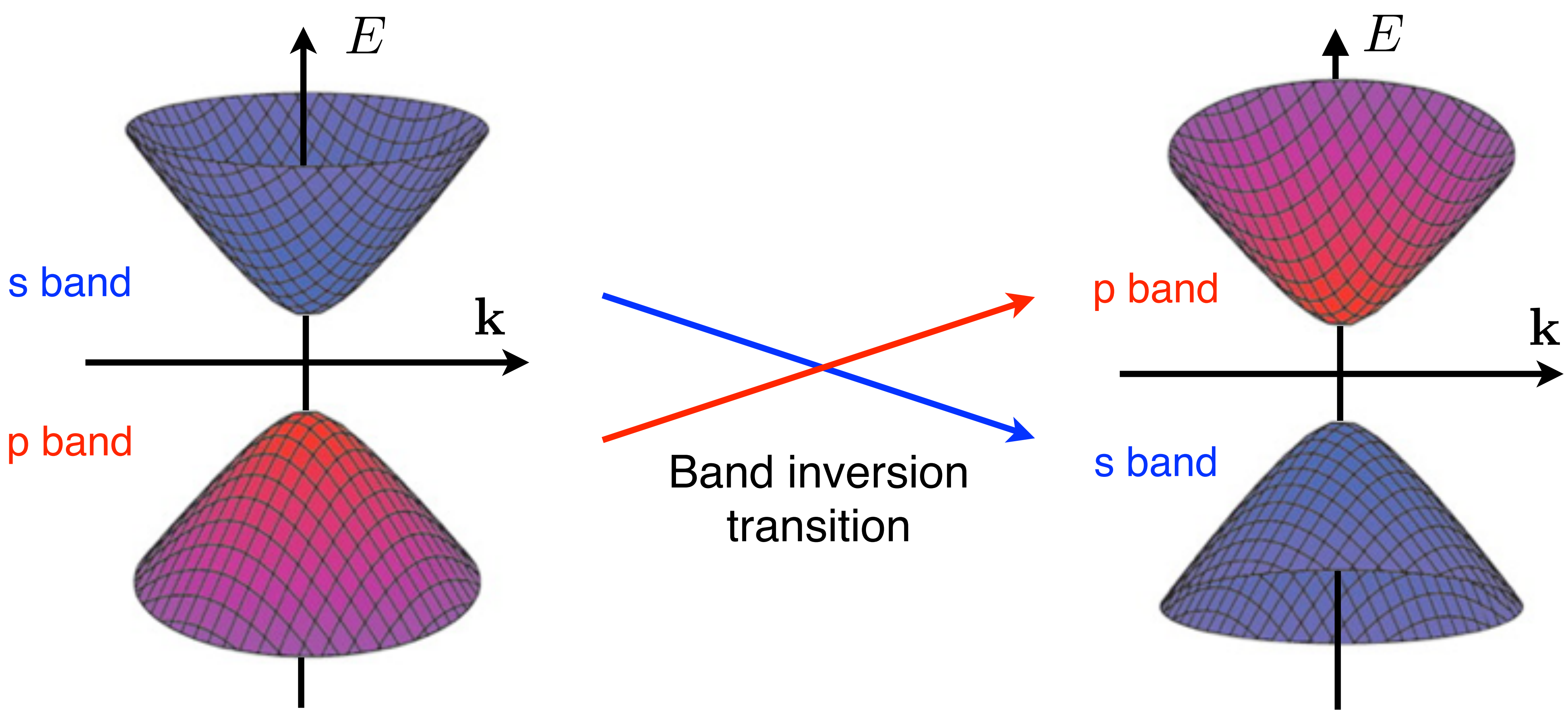
  [Bernevig, Hughes, Zhang Science 2006]

  [M. Koenig, Buhmann, Mohlenkamp, et al., Science 2007]



d < 6.3 nm: Normal band order

d > 6.3 nm: Inverted band order



$\nu = 0$: conventional insulator

$\nu = 1$: topological insulator

TRI topological insulator: HgTe quantum wells

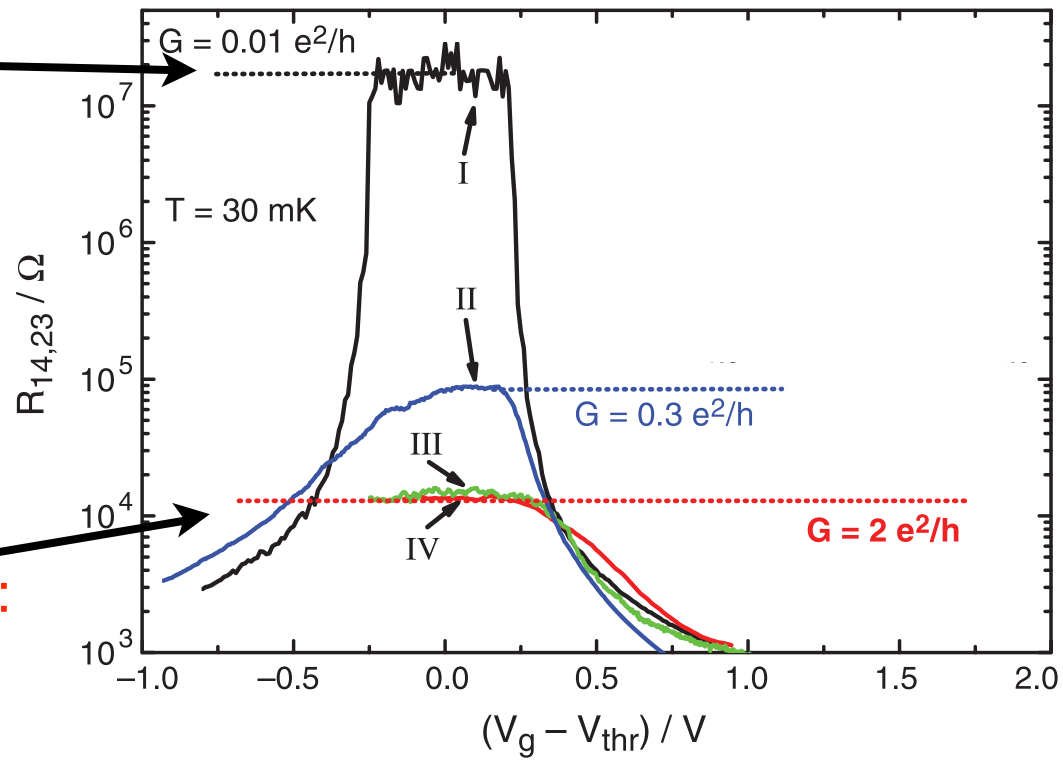
[M. Koenig, Buhmann, Mohlenkamp, et al., Science 2007]

- observed in HgTe/(Hg,Cd) quantum wells

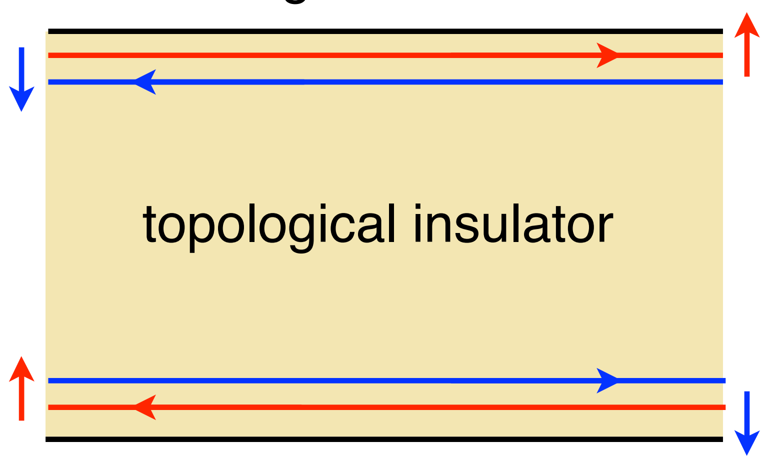
Measured conductance: $2e^2/h$ for short samples L < L_{mag}, L_{IS}
(two terminal conductance)

d < 6.3 nm
normal band order:
trivial

d > 6.3 nm
inverted band order:
topological



helical edge states:



Helical edge states are unique 1D electron conductor

- spin and momentum are locked
- no elastic backscattering from non-magnetic impurities
- perfect spin conductor!

2D topological insulator: Edge Z_2 invariant

[Kane Mele 05]

Time-reversal invariant insulators with $\Theta^2 = -1$ are classified by a **Z_2 topological invariant** ($\nu = 0,1$)

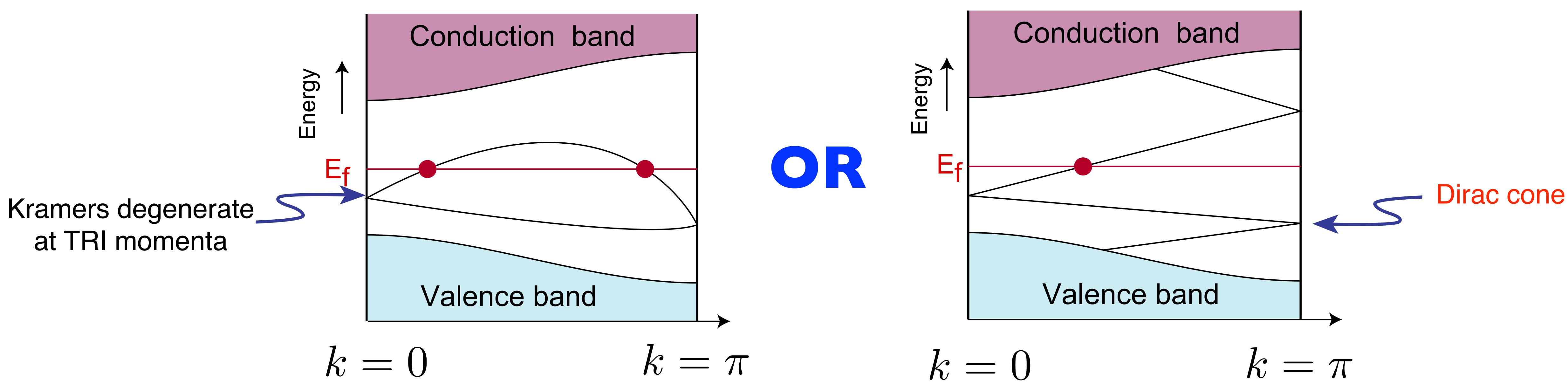
$$\boxed{\Theta \mathcal{H}(\mathbf{k}) \Theta^{-1} = +\mathcal{H}(-\mathbf{k})}$$

This can be understood via the bulk-boundary correspondence:

$\Rightarrow$ consider edge states in half of the edge Brillouin zone (other half is related by TRS)

Edge Z_2 invariant:

$\nu = 0$: conventional insulator $\qquad \nu = 1$: topological insulator



trivial phase
even # Dirac cones

non-trivial phase
odd # Dirac cones

$\Rightarrow$ **Edge Z_2 invariant** distinguishes between even / odd number of Kramers pairs of edge states

[after Hasan & Kane, RMP 2010]

2D topological insulator: First bulk Z_2 invariant

[Kane Mele 05]

[Fu and Kane]

Bulk Z_2 invariant as an obstruction to define a "TR-smooth gauge":

– $|u_n^{(1)}(\mathbf{k})\rangle$ and $|u_n^{(2)}(\mathbf{k})\rangle$ denote gauge choices in the two EBZs
– TR-smooth gauge: $|u_n^{(1)}(-\mathbf{k})\rangle = \Theta|u_n^{(2)}(\mathbf{k})\rangle$

$\Longrightarrow$ consider anti-symmetric "t-*matrix*":

$$\boxed{t_{mn}(\mathbf{k}) = \langle u_m^-(\mathbf{k})|\,\Theta\,|u_n^-(\mathbf{k})\rangle}$$

antisymmetry property: $t^{\mathrm{T}}(\mathbf{k}) = -t(\mathbf{k})$

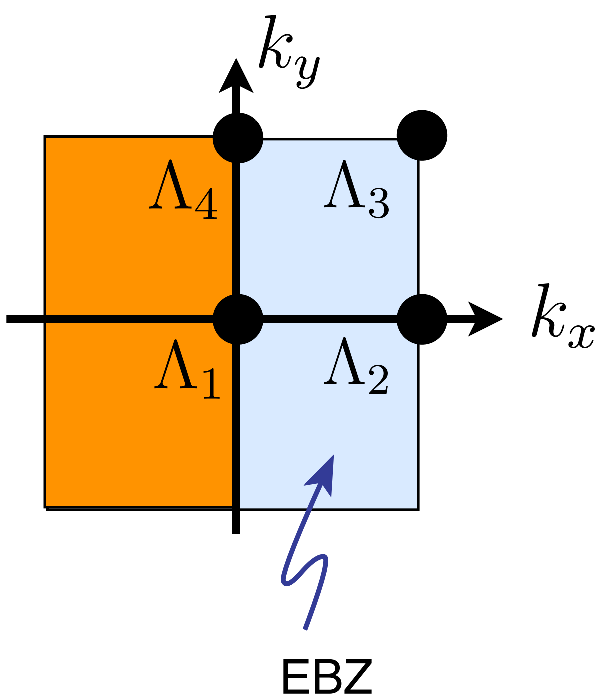
$\Longrightarrow$ Pfaffian can be defined: $\mathrm{Pf}\,[t(\mathbf{k})]$

e.g.: $\mathrm{Pf}\begin{pmatrix} 0 & z \\ -z & 0 \end{pmatrix} = z$

$$(\mathrm{Pf}\,[\omega(\Lambda_a)])^2 = \det\,[\omega(\Lambda_a)]$$

▶ Topological index counts the number or zeroes of $\mathrm{Pf}\,[t(\mathbf{k})]$ in EBZ:

$$\boxed{I = \frac{1}{2\pi i}\int_{\partial(\mathrm{EBZ})} d\mathbf{k}\cdot\nabla\log\left(\mathrm{Pf}\left[\langle u_m^-(\mathbf{k})|\Theta|u_n^-(\mathbf{k})\rangle\right]\right) \mod 2}$$



It follows from **bulk-boundary correspondence**: edge Z_2 invariant = bulk Z_2 invariant

2D topological insulator: Second bulk Z_2 invariant

[Kane Mele 05]

[Fu and Kane]

Bulk Z_2 invariant as an obstruction to define a "TR-smooth gauge":

- $|u_n^{(1)}(\mathbf{k})\rangle$ and $|u_n^{(2)}(\mathbf{k})\rangle$ denote gauge choices in the two EBZs
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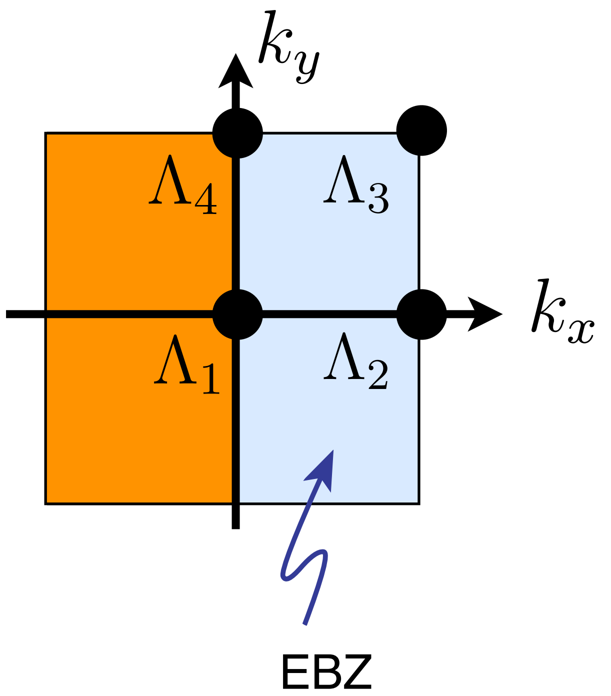


$\Rightarrow$ consider unitary *sewing matrix:*

$$\boxed{\omega_{mn}(\mathbf{k}) = \langle u_m^-(-\mathbf{k})|\Theta|u_n^-(\mathbf{k})\rangle}$$

antisymmetry property: $\omega^T(\mathbf{k}) = -\omega(-\mathbf{k})$

at TRI momenta: $\Lambda_a = -\Lambda_a \Rightarrow \omega^T(\Lambda_a) = -\omega(\Lambda_a)$ is antisymmetric

$\Rightarrow$ Pfaffian can be defined: $\mathrm{Pf}\,[\omega(\Lambda_a)]$ e.g.: $\mathrm{Pf}\begin{pmatrix} 0 & z \\ -z & 0 \end{pmatrix} = z$

$$(\mathrm{Pf}\,[\omega(\Lambda_a)])^2 = \det\,[\omega(\Lambda_a)]$$

Bulk Z_2 invariant (ν= 0,1):

$$\boxed{(-1)^\nu = \prod_{a=1}^{4} \frac{\mathrm{Pf}\,[\omega(\Lambda_a)]}{\sqrt{\det\,[\omega(\Lambda_a)]}} = \pm 1}$$

(gauge invariant, but smooth gauge needed)

It follows from **bulk-boundary correspondence**: edge Z_2 invariant = bulk Z_2 invariant

2D topological insulator: Bulk Z_2 invariants

Three equivalent definitions for **bulk Z_2 topological invariant:**

(A) in terms of sewing matrix:

$$\boxed{(-1)^{\nu} = \prod_{a=1}^{4} \frac{\mathrm{Pf}\left[\omega(\Lambda_a)\right]}{\sqrt{\det\left[\omega(\Lambda_a)\right]}} = \pm 1}$$

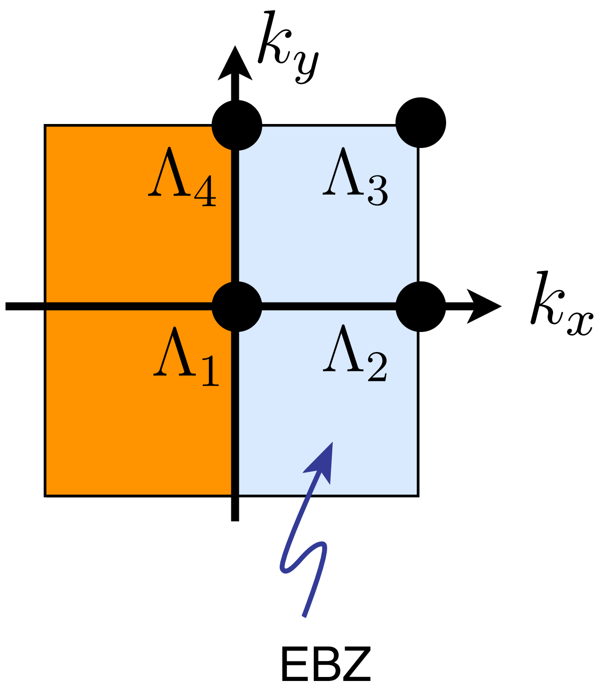
(gauge invariant, but smooth gauge needed)

sewing matrix: $\omega_{mn}(\mathbf{k}) = \langle u_m^-(-\mathbf{k})|\Theta|u_n^-(\mathbf{k})\rangle$ (is unitary, and anti-symmetric at TRI momenta)

(B) count number of zeroes of $\mathrm{Pf}\left[\langle u_m^-(\mathbf{k})|\Theta|u_n^-(\mathbf{k})\rangle\right]$ in EBZ

$$I = \frac{1}{2\pi i}\int_{\partial(\mathrm{EBZ})} d\mathbf{k}\cdot\nabla\log\left(\mathrm{Pf}\left[\underbrace{\langle u_m^-(\mathbf{k})|\Theta|u_n^-(\mathbf{k})\rangle}\right]\right) \mod 2$$

(antisymmetric at all momenta, but not unitary)



(C) in terms of Berry connection:

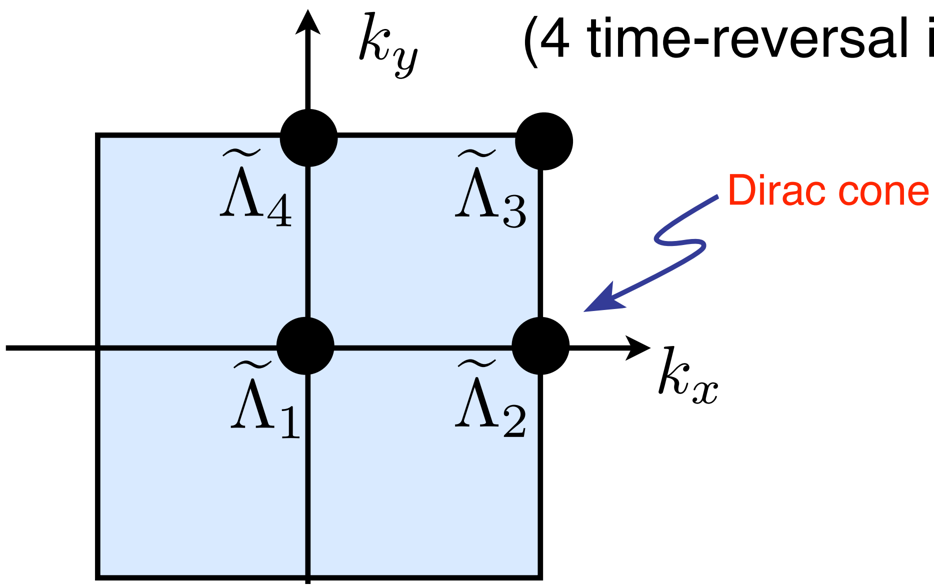
$$\nu = \frac{1}{2\pi}\left[\oint_{\partial(EBZ)} d\boldsymbol{k}\cdot\mathcal{A} - \int_{\mathrm{EBZ}} d^2\boldsymbol{k}\,\mathcal{F}\right] \mod 2$$

3D topological insulator: Surface Z_2 invariant

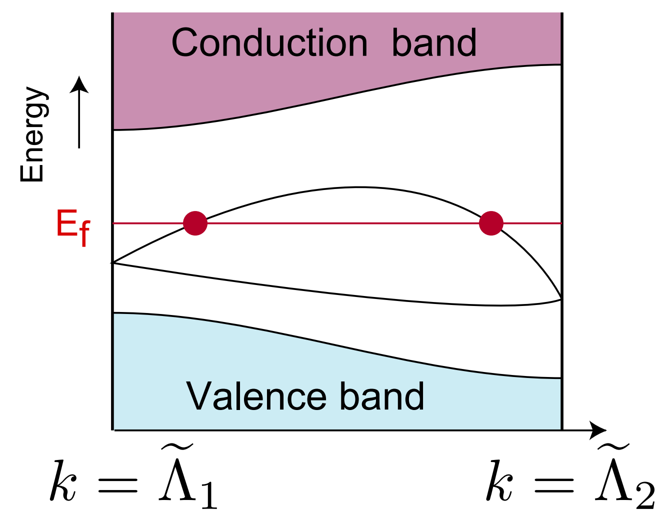
- How do surface states connect between TRI momenta?

[after Hasan & Kane, RMP 2010]

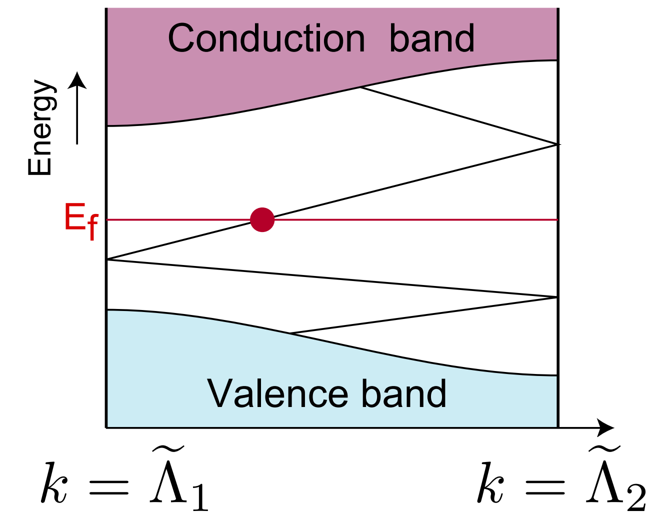
(4 time-reversal invariant momenta in surface BZ)



surface Brillouin zone



OR



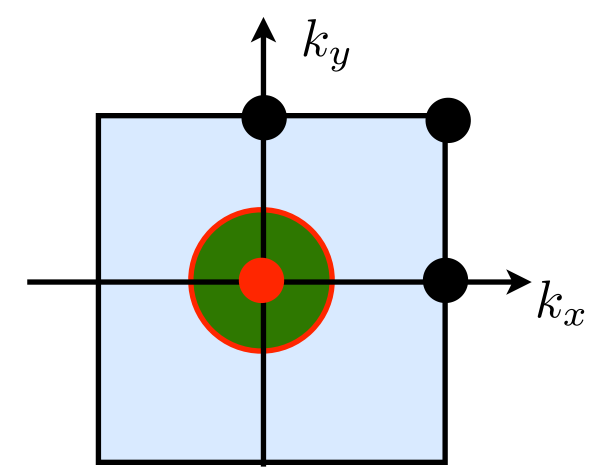
- **Surface Z_2 invariant:**

$\nu = 1$: **Strong topological insulator**

— Fermi surface encloses *odd* number of TRI momenta

— independent of surface orientation

— protected by time-reversal symmetry

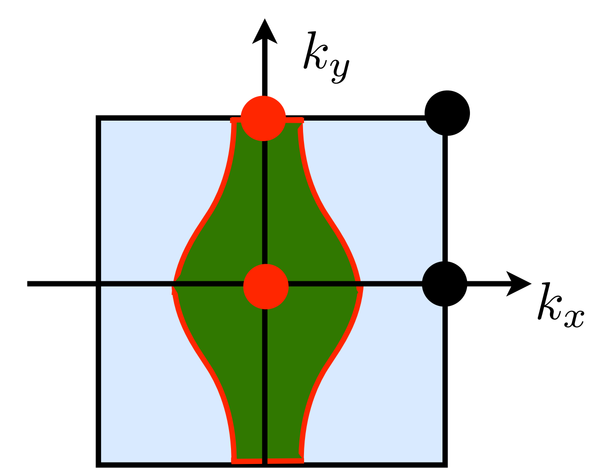


$\nu = 0$: **Weak topological insulator**

— Fermi surface encloses *even* number of TRI momenta

— depends on surface orientation (quasi-2D topological insulator)

— protected by time-reversal *and* translation symmetry

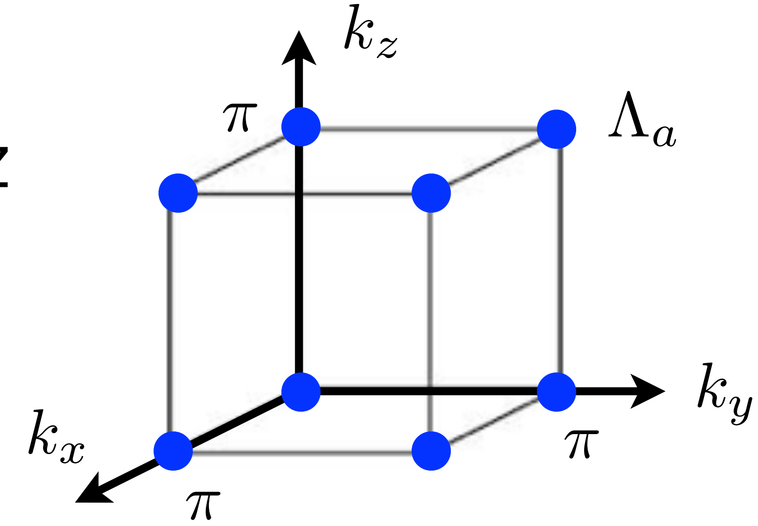


3D topological insulator: Bulk Z_2 invariant

- **Bulk Z_2 invariant:**

[Kane-Mele, Moore-Balents, Roy, Fu-Kane-Mele (06-07)]

8 TRI momenta in bulk BZ

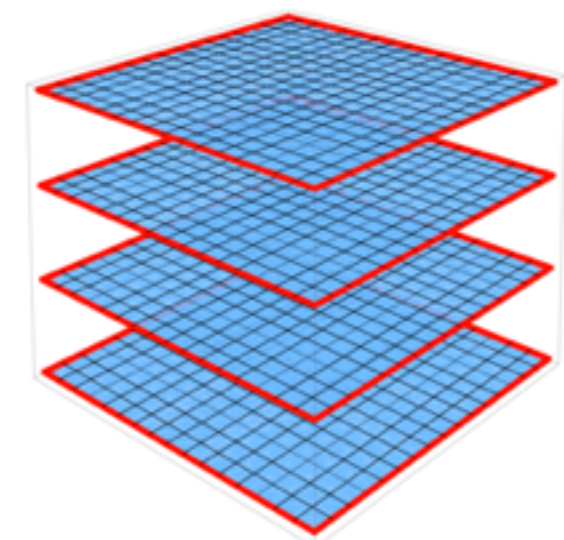


  – Strong Z_2 invariant

$$(-1)^{\nu} = \prod_{a=1}^{8} \frac{\mathrm{Pf}\left[\omega(\Lambda_a)\right]}{\sqrt{\det\left[\omega(\Lambda_a)\right]}} = \pm 1$$

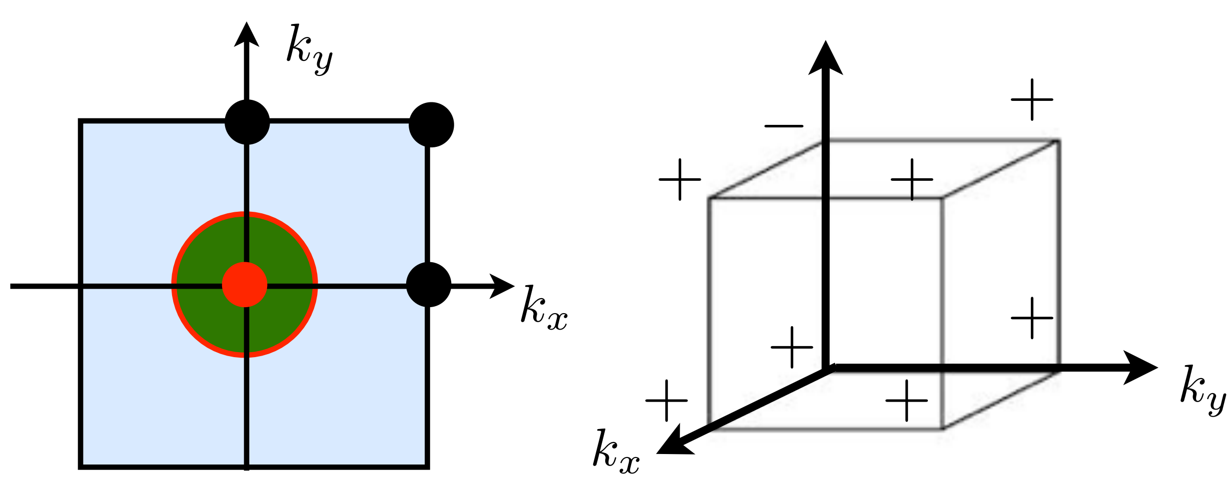
  – Weak Z_2 invariant

$$(-1)^{\nu_i} = \prod_{a=1}^{4} \frac{\mathrm{Pf}\left[\omega(\Lambda_a)\right]}{\sqrt{\det\left[\omega(\Lambda_a)\right]}}\Bigg|_{k_i=0}$$

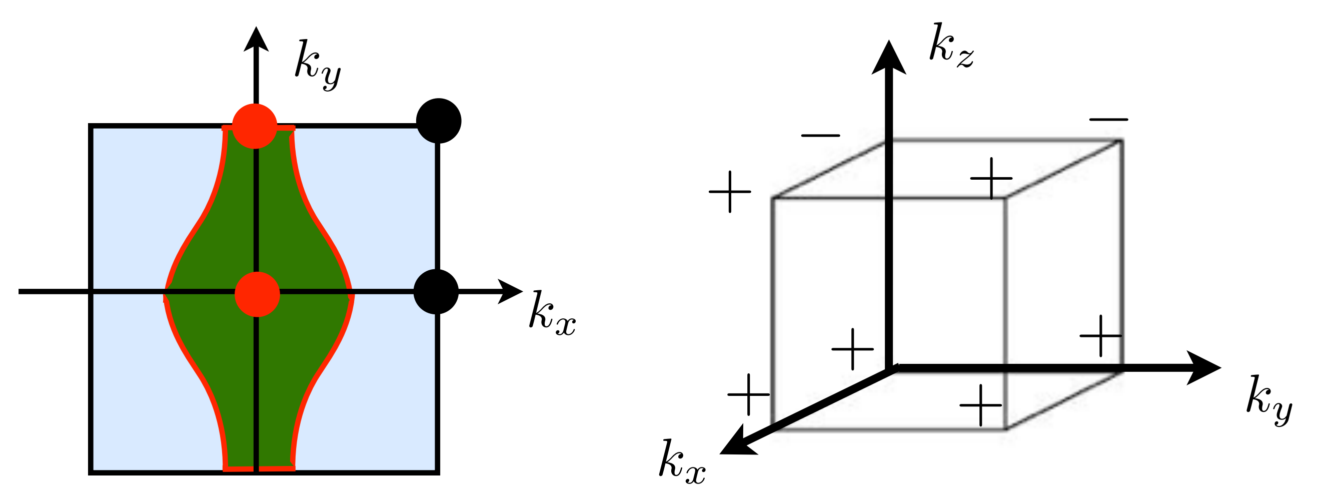


Bulk-boundary correspondence: edge Z_2 invariant = bulk Z_2 invariant

Strong topological insulator



Weak topological insulator



Experimental detection of 3D topological insulators

▶ observed in certain band insulators with strong spin-orbit coupling

BiSb alloy, Bi_2Se_3, Bi_2Te_3, $TlBiTe_2$, $TlSbSe_2$, etc ....

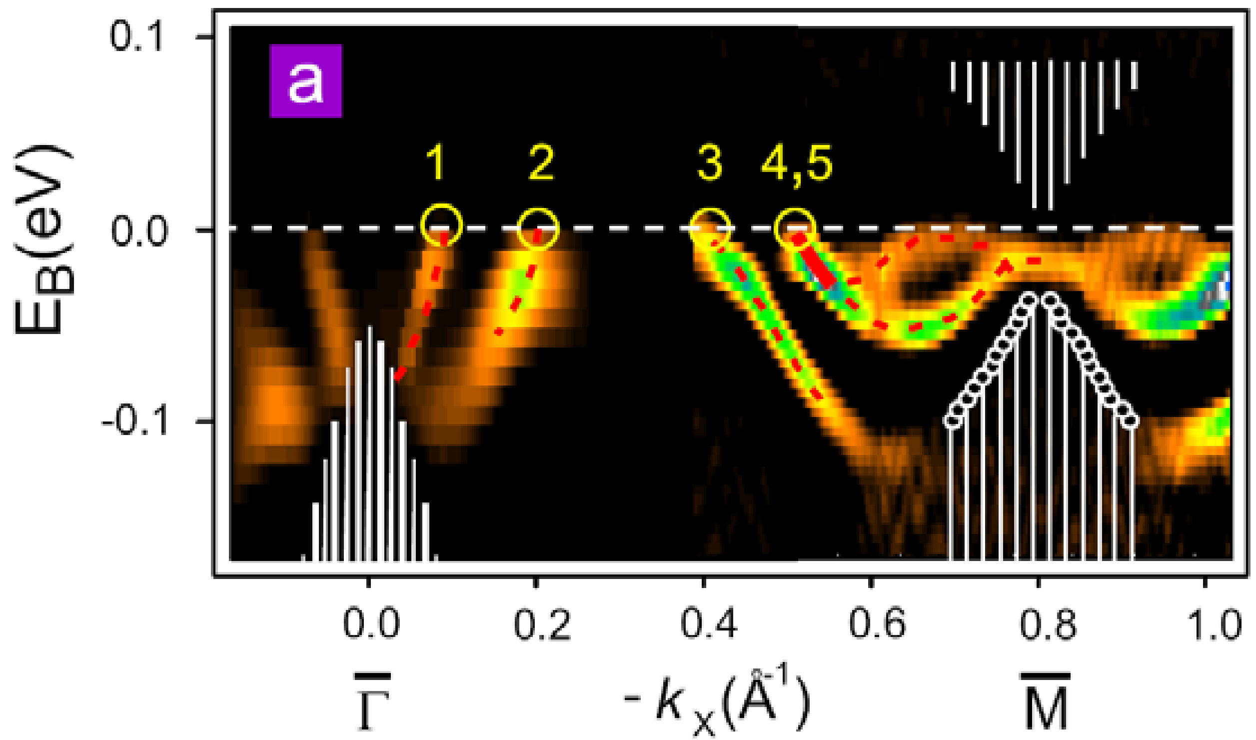
stable surface states cross a gap, that is opened up by spin-orbit coupling

- $\mathrm{Bi}_{1-x}\ \mathrm{Sb}_x$:

[Fu, Kane, PRL 2007]

[Hsieh, Hasan et al, Nature 2008]

momentum resolved photoemission (ARPES)



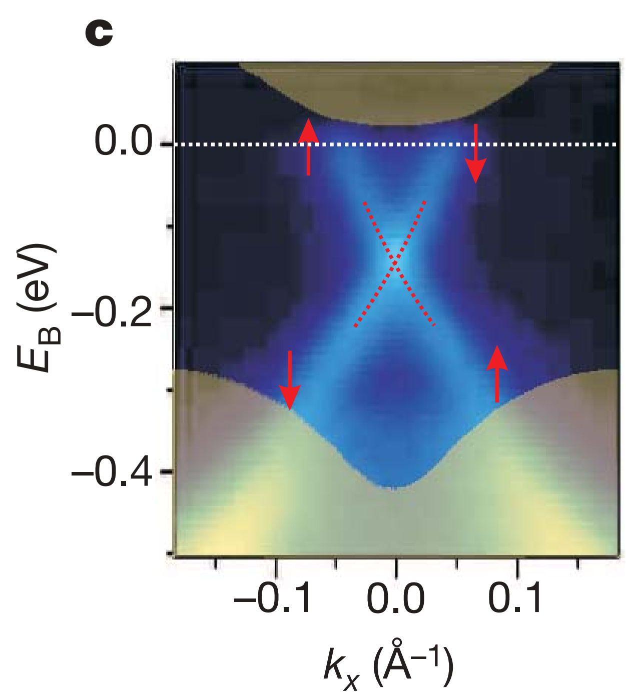
five surface state bands cross E_F between TRI momenta $\bar{\Gamma}$ and $\bar{\mathrm{M}}$

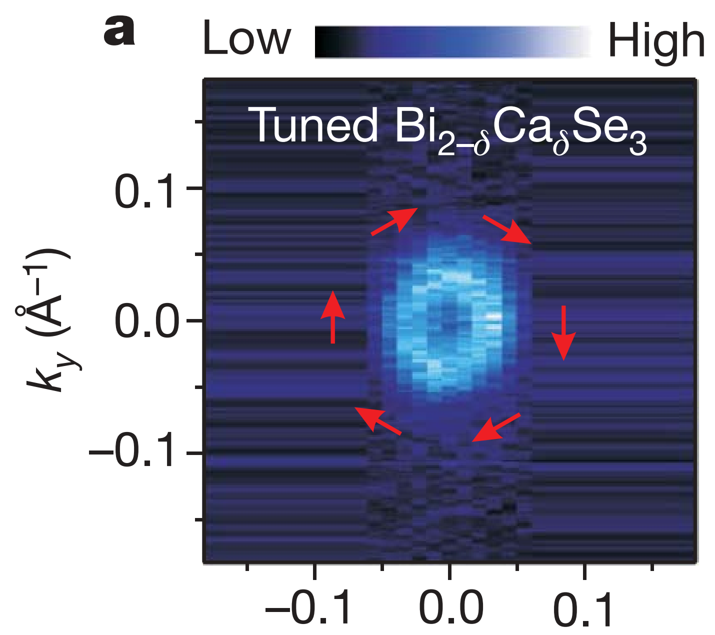
$\Rightarrow$ strong topological insulator

Experimental detection of 3D topological insulators

- $Bi_2\ Se_3$:

spin resolved and momentum resolved photoemission (ARPES)

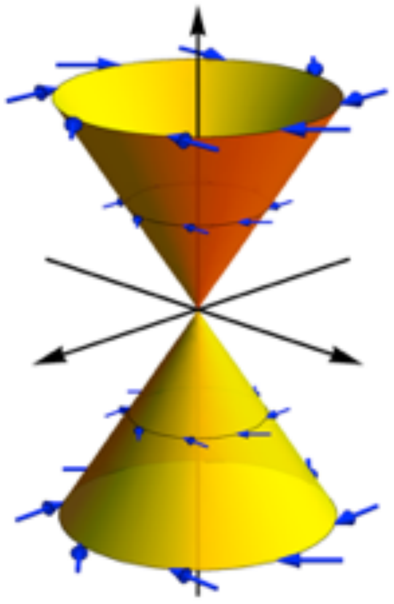




[H. Zhang et al., Nat Phys 2009]

[Hsieh, Hasan et al, Nature 2009]

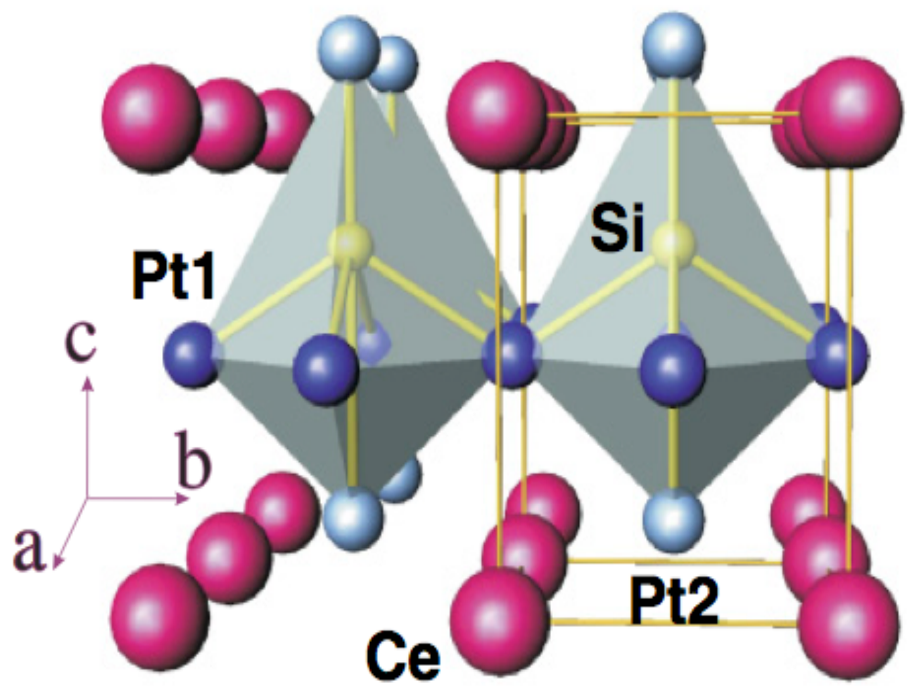
simple surface state structure, similar to graphene



Unique properties of helical surface states:

- spin and momentum are locked
- half of an ordinary 2DEG, "1/4 of graphene"
- robust to disorder, impossible to localize

Topological Superconductors



Bogoliubov-de Gennes theory for superconductors

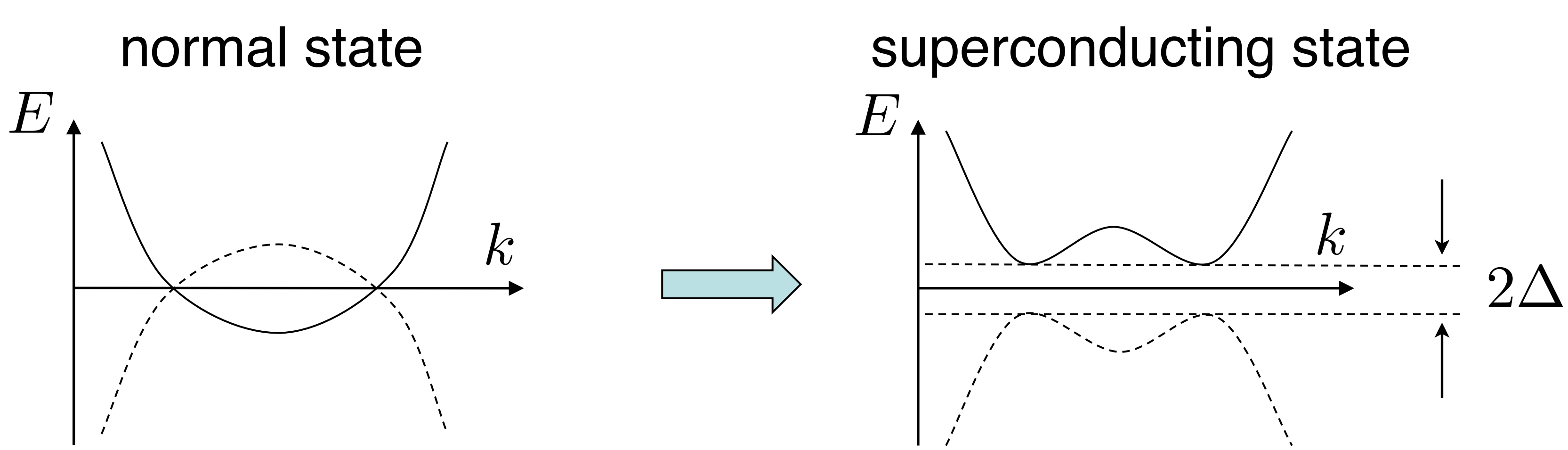
Superconductor = Cooper pairs (boson) + Bogoliubov quasiparticles (fermions)

BCS mean field theory: $c^\dagger c\, c^\dagger c \quad \Rightarrow \quad \langle c^\dagger c^\dagger \rangle c\, c = \Delta^* c\, c$

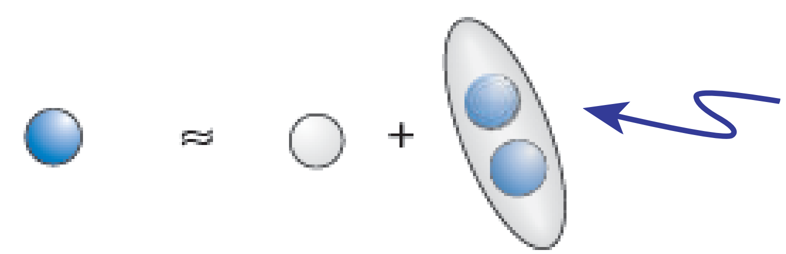
$$H = \frac{1}{2} \sum_{\boldsymbol{k}} \begin{pmatrix} c^\dagger & c \end{pmatrix} H_{\mathrm{BdG}} \begin{pmatrix} c \\ c^\dagger \end{pmatrix}$$

Bogoliubov-de Gennes Hamiltonian

$$H_{\mathrm{BdG}} = \begin{pmatrix} h_0 & \Delta \\ \Delta^\dagger & -h_0^T \end{pmatrix}$$



Built-in anti-unitary particle-hole symmetry:



$$\boxed{\Xi\, \mathcal{H}_{\mathrm{BdG}}(\boldsymbol{k})\, \Xi^{-1} = -\mathcal{H}_{\mathrm{BdG}}(-\boldsymbol{k})} \qquad \Xi = \tau_x \mathcal{K} \qquad \Xi^2 = +1 \qquad \tau_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

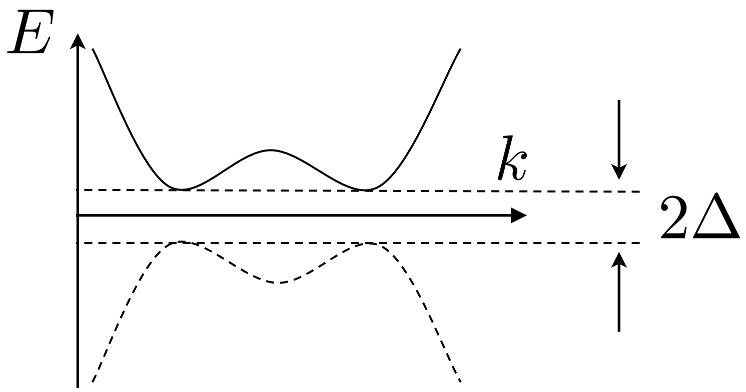
(for triplet pairing)

Particle-hole redundancy $\quad \Xi\, \psi_{+\boldsymbol{k},+E} = \tau_x \psi^*_{-\boldsymbol{k},-E} \quad \Rightarrow \quad \gamma^\dagger_{\boldsymbol{k},E} = \gamma_{-\boldsymbol{k},-E}$

Topological superconductors

Bogoliubov-de Gennes Hamiltonian

$$H_{\mathrm{BdG}} = \begin{pmatrix} h_0 & \Delta \\ \Delta^\dagger & -h_0^T \end{pmatrix}$$

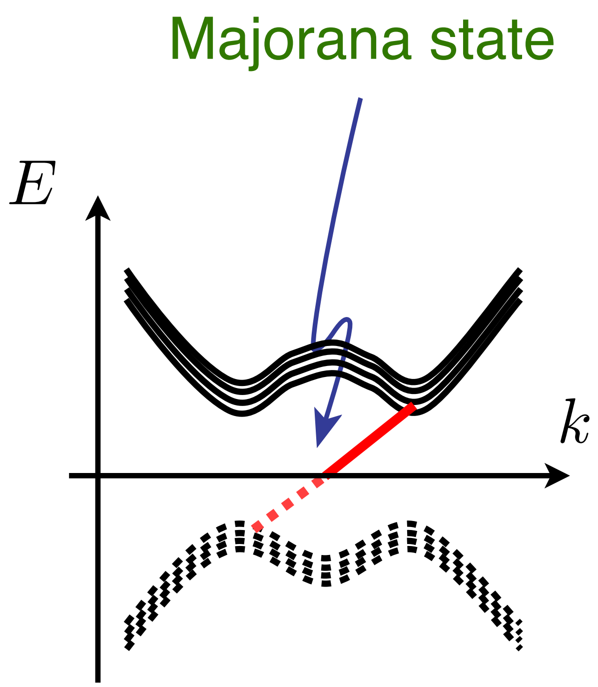


gap in spectrum ⟶ can define top. invariant

$$n = \frac{i}{2\pi} \int_{\substack{\text{states} \\ \text{with } E<0}} \mathcal{F}\, d\boldsymbol{k}$$

BdG band structures are equivalent if they can be continuously deformed into one another without closing the energy gap and without breaking the symmetries of the SC.

Symmetries to consider: Particle-hole symmetry, time-reversal, etc.



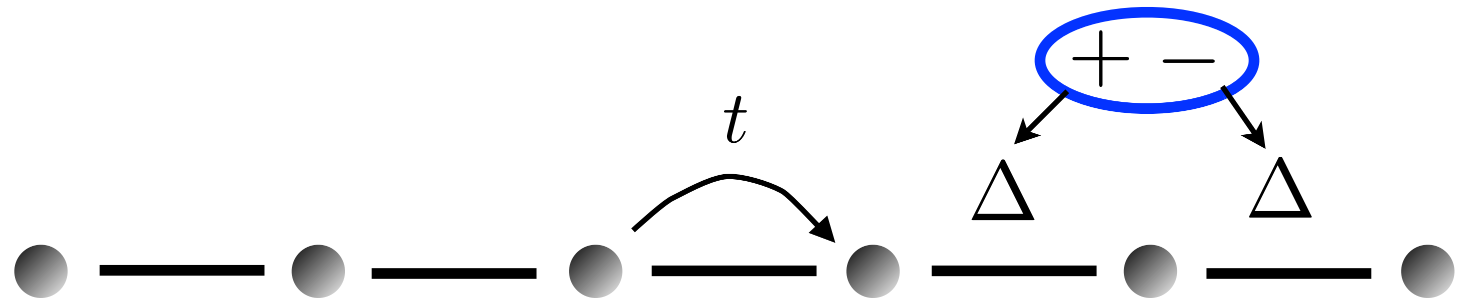
Bulk-boundary correspondence + particle-hole redundancy:

$$\Xi\, \psi_{+\boldsymbol{k},+E} = \tau_x \psi^*_{-\boldsymbol{k},-E}$$

- Majorana edge state at zero energy

1D topological superconductor: Majorana chain

One-dimensional spinless p-wave superconductor: Majorana chain



Hamiltonian: $$\mathcal{H} = \sum_j \left[t(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j - \mu c_j^\dagger c_j + \Delta(c_{j+1}^\dagger c_j^\dagger + c_j c_{j+1}) \right]$$

in momentum space: $$\mathcal{H} = \frac{1}{2} \sum_k \begin{pmatrix} c_k^\dagger & c_{-k} \end{pmatrix} \mathcal{H}_{\mathrm{BdG}}(k) \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}$$

$$\mathcal{H}_{\mathrm{BdG}}(k) = \mathbf{d}(k) \cdot \vec{\tau}$$

$$d_x(k) = \Delta \sin k \qquad d_y(k) = 0$$

$$d_z(k) = 2t \cos k - \mu$$

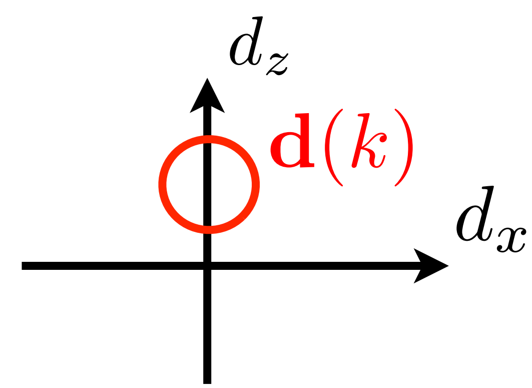
Particle-hole symmetry:

$$\tau_x \mathcal{H}^*_{\mathrm{BdG}}(k) \tau_x = -\mathcal{H}_{\mathrm{BdG}}(-k)$$

Time-reversal symmetry:

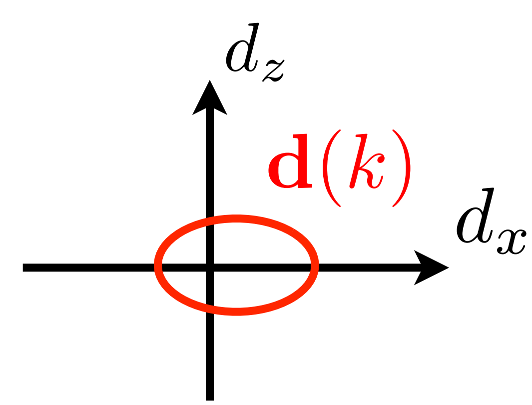
$$\tau_z \mathcal{H}^*_{\mathrm{BdG}}(k) \tau_z = +\mathcal{H}_{\mathrm{BdG}}(-k)$$

energy spectrum: $E_\pm = \pm |\mathbf{d}(k)|$



$|\mu| > 2t$:

trivial superconductor



$|\mu| < 2t$:

topological superconductor

1D topological superconductor: Majorana chain

[Kitaev 2000]

To reveal zero-energy edge states, consider different viewpoint: **Majorana representation**

> Majorana fermion:
> Particle = Antiparticle

$$c_j = \frac{1}{2}\left(\gamma_{1j} + i\gamma_{2j}\right) \qquad c_j^\dagger = \frac{1}{2}\left(\gamma_{1j} - i\gamma_{2j}\right)$$

Commutation relations: $\left\{\gamma_{lj}, \gamma_{l'j'}\right\} = 2\delta_{ll'}\delta_{jj'} \qquad \left(\gamma_{lj}\right)^2 = 1$

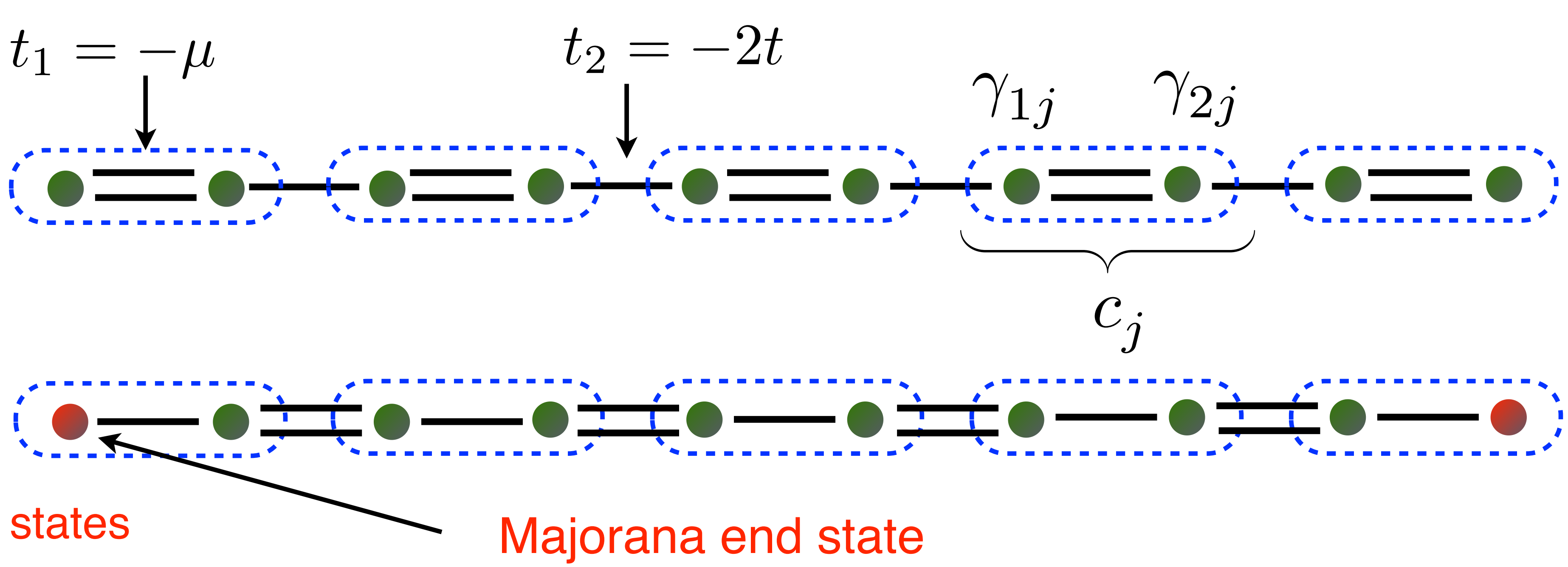
$\Longrightarrow$ Majorana chain for spinless fermions

$$H = \frac{i}{2}\sum_j \left[-\mu\gamma_{1j}\gamma_{2j} + (\Delta - t)\gamma_{2j}\gamma_{1j+1} + (\Delta + t)\gamma_{1j}\gamma_{2j+1}\right]$$

for $\Delta = -t$: nearest neighbor Majorana chain

$t_1 > t_2$:
trivial superconductor

$t_1 < t_2$:
topological superconductor

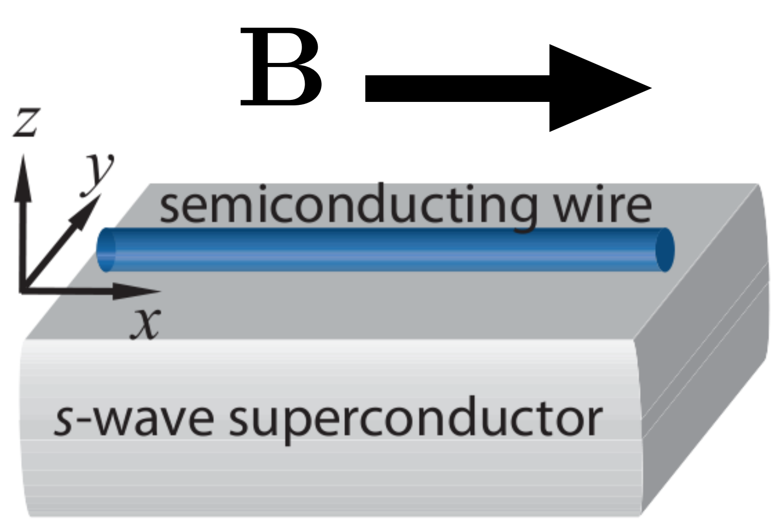


— zero-energy Majorana end states

Experimental detection of 1D spinless topological SC

1D spinless chiral p-wave superconductor is likely (?) realized in InSb-nanowire-heterostructures

magnetic field B

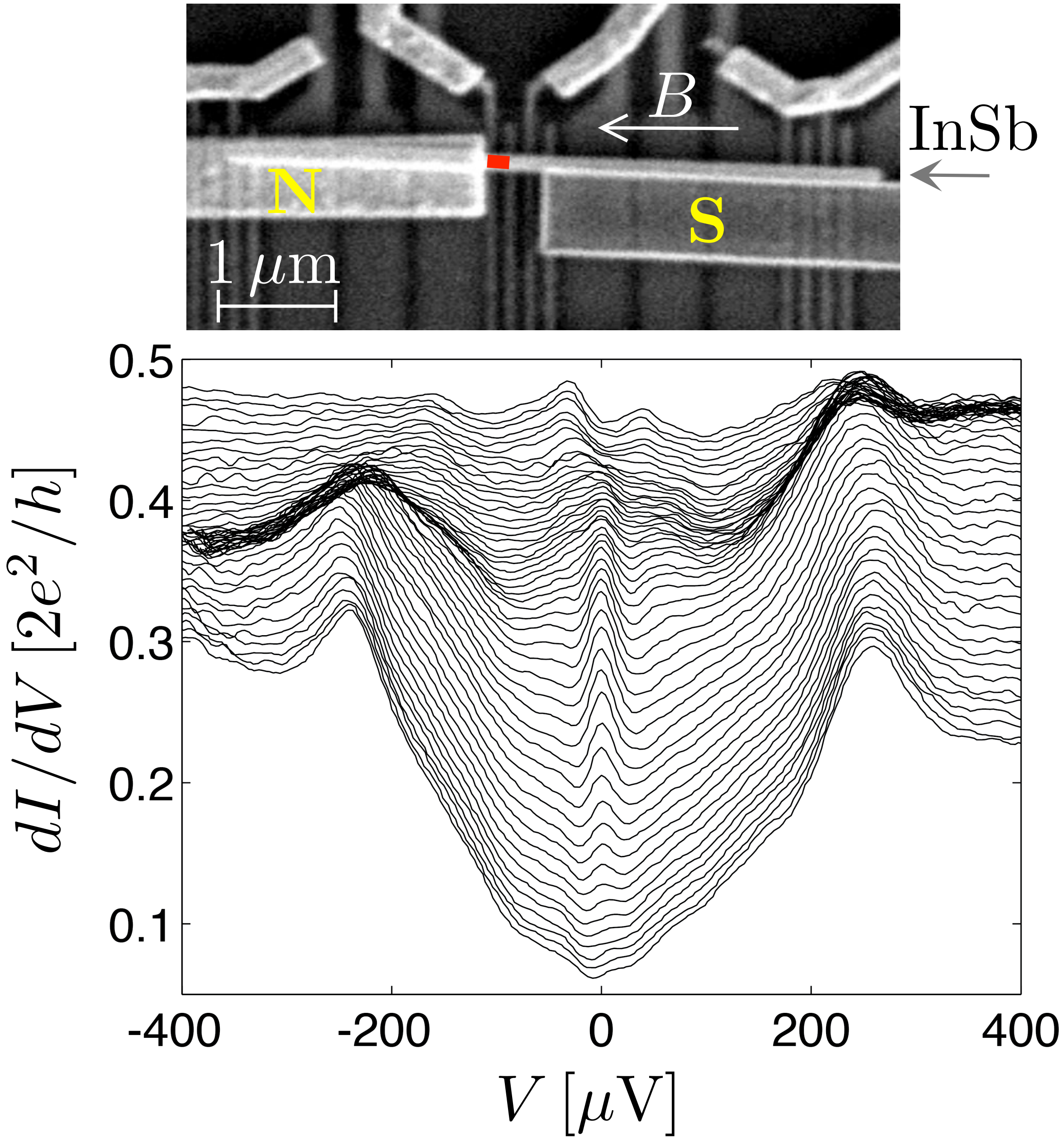


- Condition for topological phase:

$$B \propto E_{\text{Zeeman}} > \sqrt{\Delta^2 - \mu^2}$$

[Sau, Lutchyn, Tewari, das Sarma, et al 2009]
[Oreg, von Oppen, et al 2010]

[after Alicea, Rep. Prog. Phys. 2012]



Differential tunneling conductance as a function of magnetic field B

[Mourik, Kouwenhoven et al, Science 2012]

Two-dimensional spinless chiral p-wave SC

Lattice BdG model: **[Read & Green 00]**

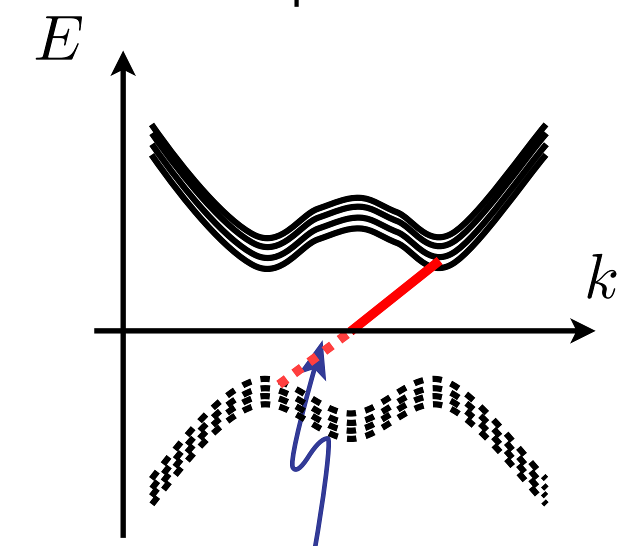
$$\mathcal{H}_{\mathrm{BdG}} = \left(2t\left[\cos k_x + \cos k_y\right] - \mu\right)\tau_z + \Delta_0\left(\tau_x \sin k_x + \tau_y \sin k_y\right) = \boldsymbol{m}(\boldsymbol{k}) \cdot \boldsymbol{\tau}$$

(similar to Chern insulator, but different symmetries,
different physical interpretation)

Particle-hole symmetry: $\tau_x \mathcal{H}^*_{\mathrm{BdG}}(\mathbf{k})\tau_x = -\mathcal{H}_{\mathrm{BdG}}(-\mathbf{k})$

$E = \pm\left|\boldsymbol{m}(\boldsymbol{k})\right|$ Spectrum flattening: $\hat{\boldsymbol{m}}(\boldsymbol{k}) = \dfrac{\boldsymbol{m}(\boldsymbol{k})}{|\boldsymbol{m}(\boldsymbol{k})|}$

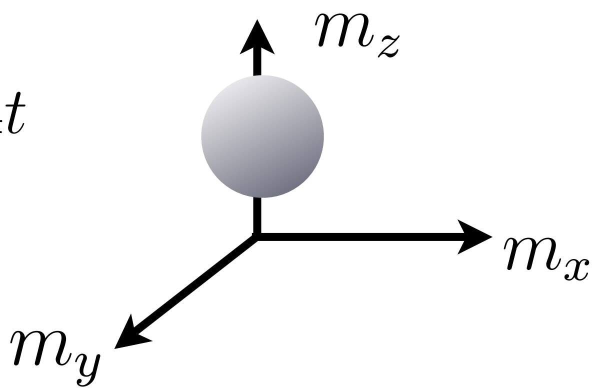
edge band structure
of chiral p-wave SC



trivial phase

$|\mu| > 4t$

$n = 0$

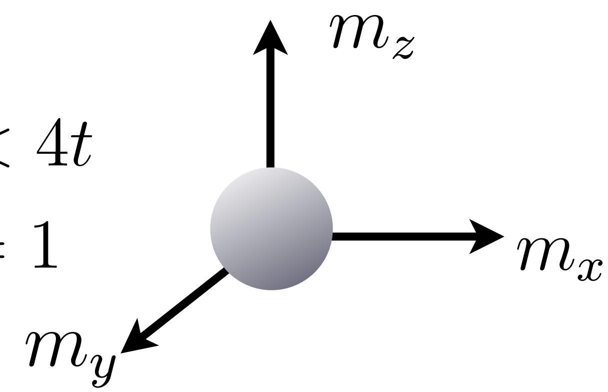


no edge state

non-trivial phase

$|\mu| < 4t$

$n = 1$



chiral Majorana edge state

classified by
Chern number:
(winding number)

$$n = \frac{1}{8\pi}\int_{\mathrm{BZ}} d^2\mathbf{k}\, \epsilon^{\mu\nu}\, \hat{\mathbf{m}} \cdot \left[\partial_{k_\mu}\hat{\mathbf{m}} \times \partial_{k_\nu}\hat{\mathbf{m}}\right]$$

Sr_2RuO_4 (n=2)

Mapping $\hat{\boldsymbol{m}}(\boldsymbol{k})$: Brillouin zone $\longmapsto$ $\hat{\boldsymbol{m}}(\boldsymbol{k}) \in S^2$ "$\pi_2(S^2) = \mathbb{Z}$"

Majorana fermions in chiral p-wave superconductor

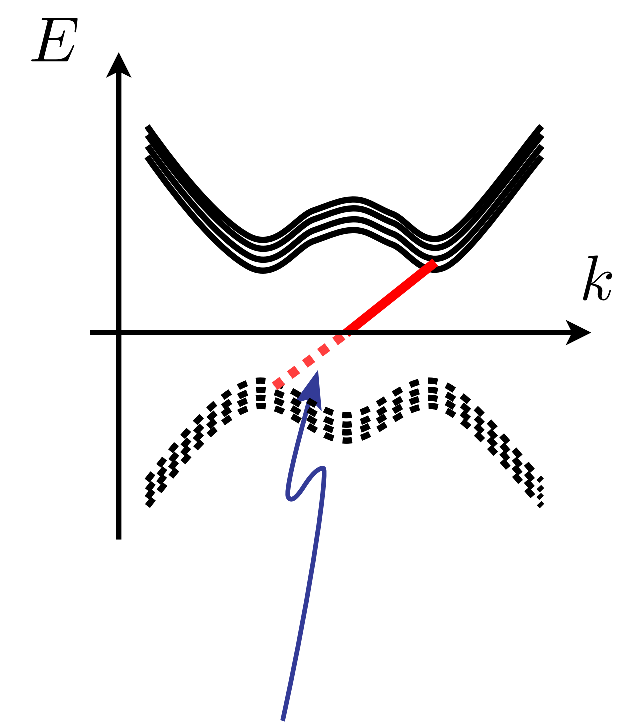
- Bulk-boundary correspondence: $n = \#$ Majorana edge modes

Majorana edge states are perfect heat conductor

⟹ Quantized thermal Hall conductance

$$\frac{\kappa_{xy}}{T} = \frac{\pi k_B^2}{48h} \int_{\mathrm{BZ}} d^2\mathbf{k}\, \epsilon^{\mu\nu} \hat{\mathbf{m}} \cdot \left[\partial_{k_\mu} \hat{\mathbf{m}} \times \partial_{k_\nu} \hat{\mathbf{m}}\right]$$

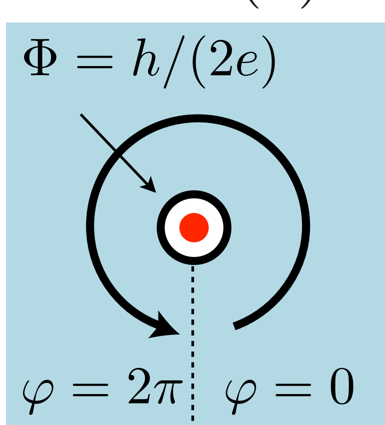
edge band structure
of chiral p-wave SC



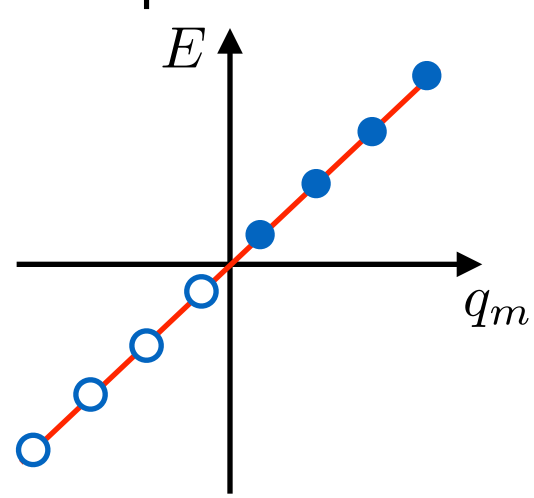
Majorana state

- Majorana zero mode at a vortex:
  - vortex: small hole with edge states
  - Majorana zero mode for $\Phi = p\dfrac{h}{2e}$ with p odd
    (periodic vs. anti-periodic BC)

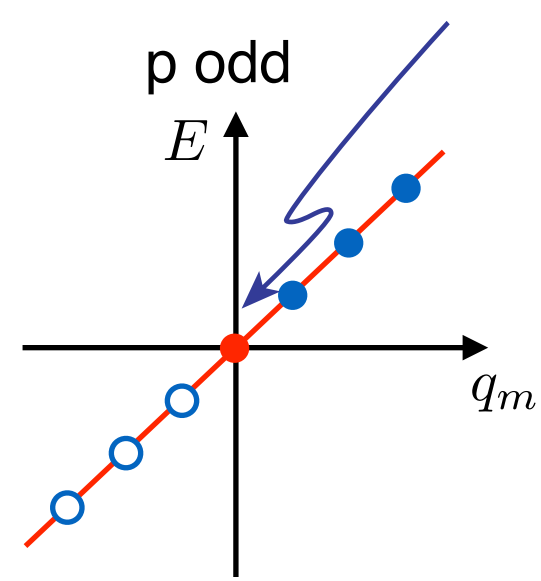
$$\Delta = \Delta(r)e^{i\varphi}$$



p even



p odd

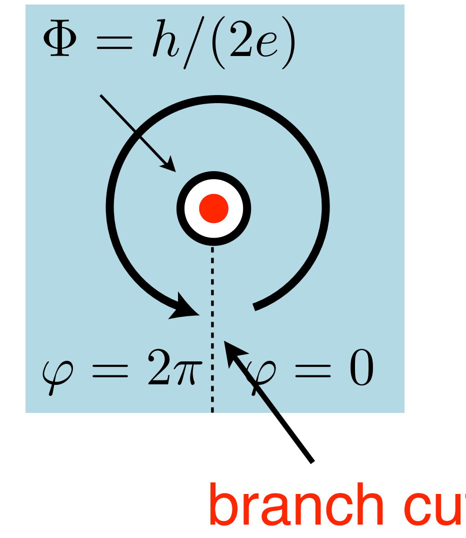


[Caroli, de Gennes, Matricon '64]

Majorana fermions for topological quantum computing

Degenerate states associated with Majorana zero modes at vortices define a topologically protected quantum memory

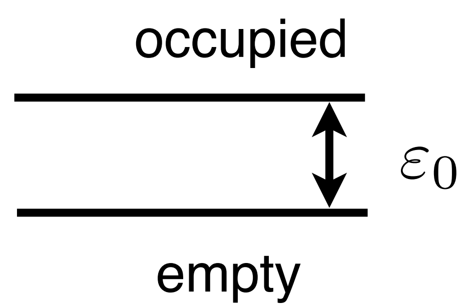
$$\Delta = \Delta(r)e^{i\varphi}$$



[Kitaev 2003]

▶ Two Majorana fermions define a single two level system:

$$H = \frac{i}{2}\varepsilon_0\gamma_2\gamma_1 = \varepsilon_0 c^\dagger c$$



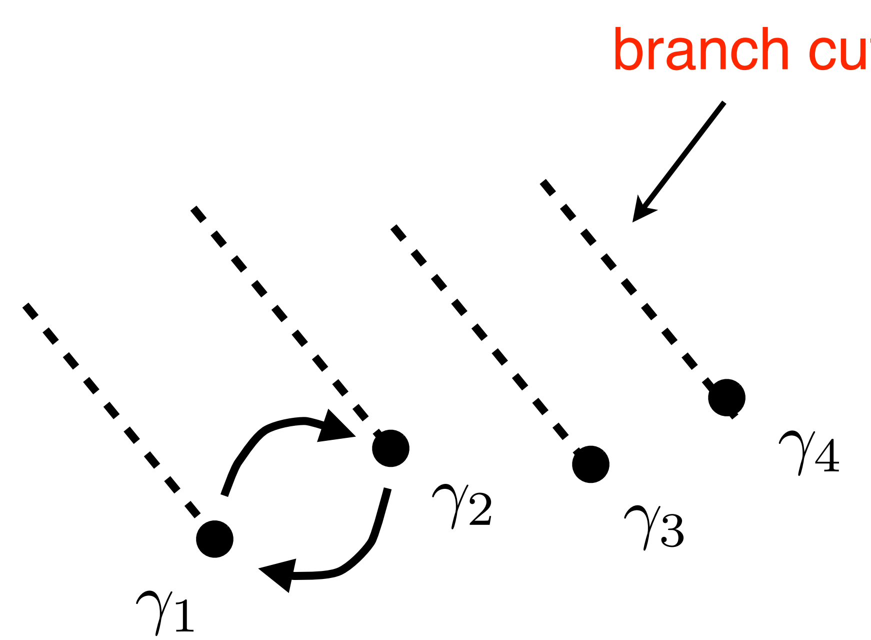
— two degenerate states (occupied / empty) $\Rightarrow$ 1 qubit

— 2N separated Majoranas corresponds to N qubits

— information stored non-locally $\Longrightarrow$ immune to local perturbations / decoherence

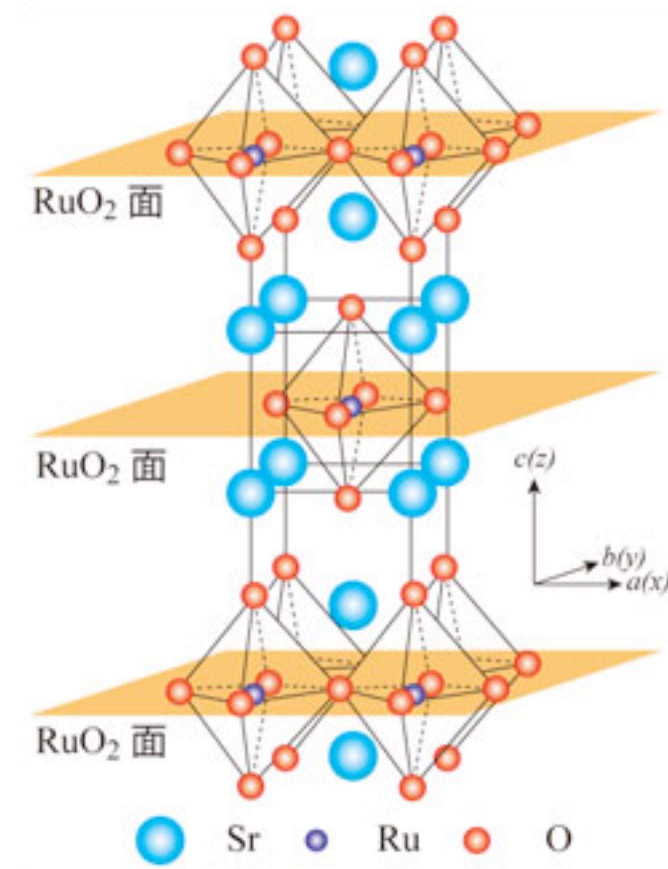
▶ Majorana vortex bound states obey non-Abelian statistics

$$\gamma_2 \rightarrow \gamma_1$$
$$\gamma_1 \rightarrow -\gamma_2$$



Experimental detection of spinful chiral p-wave SC

The transition-metal-oxide Sr_2RuO_4 is likely (?) a *spinful* chiral p-wave superconductor with Chern number n=2 (per layer)



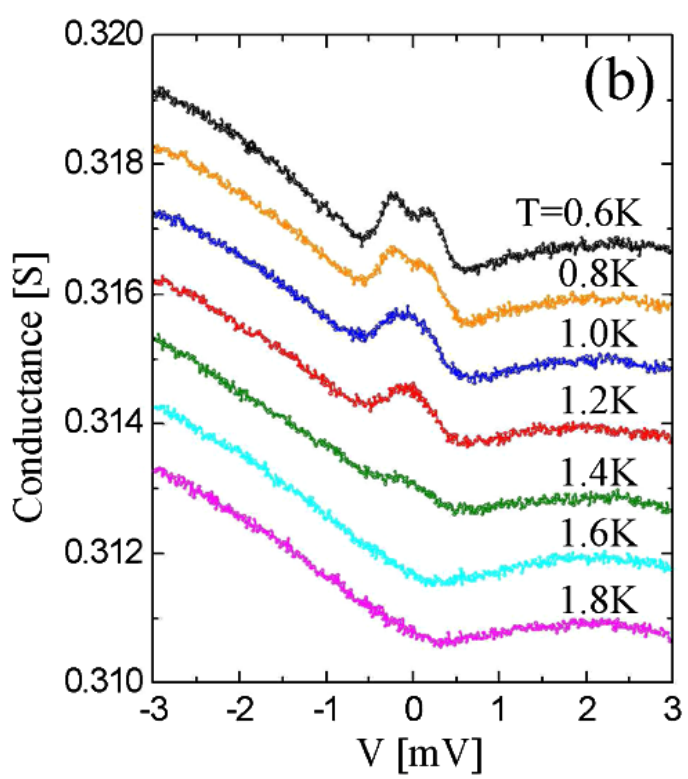
— Ru t_{2g}-orbitals ($4d^4$-electrons) hybridized with O p-oribitals form quasi-two-dimensional Fermi surfaces

— transition temperature T_C = 1.5K

— strong anisotropies in spin dependent responses (NMR and Knight shift)

— signatures of edge states in tunneling conductance

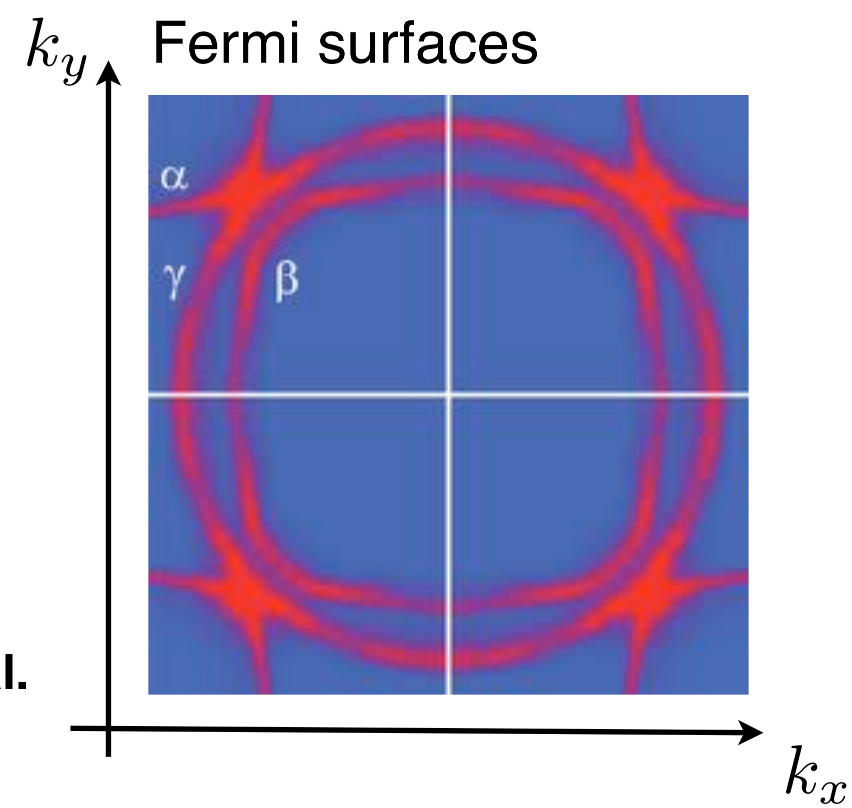
tunneling conductance



[Maeno et al. JPSJ 81, 011009]

[Kashiwaya et al. PRL 2011]

Fermi surfaces



[Damascelli et al. PRL 2000]