Nonadiabatic dynamics and coherent control of nonequilibrium superconductors

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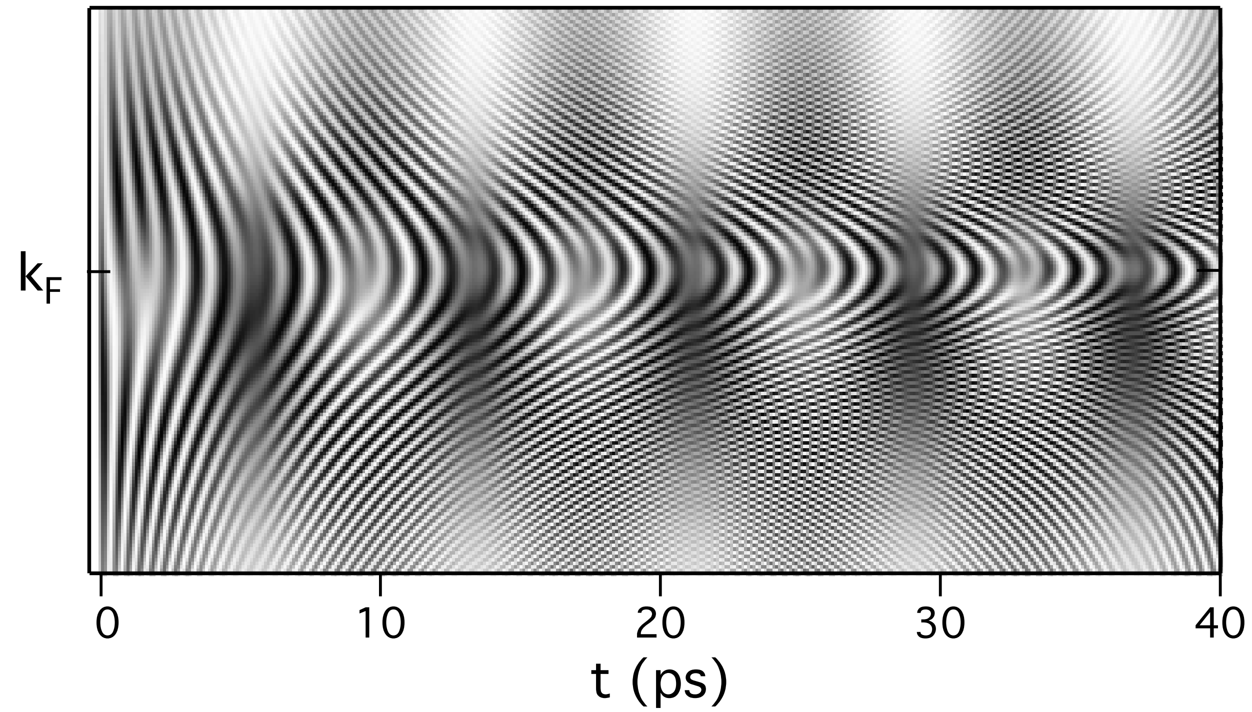
Workshop on strongly correlated electron systems

Schloss Ringberg, November 2013



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Outline

1. Introduction

- Recent pump-probe experiments on NbN films
- How does a quantum system relax?

2. Nonadiabatic dynamics of superconductors

- Superconductor coupled to laser field and optical phonons
- Density matrix formalism

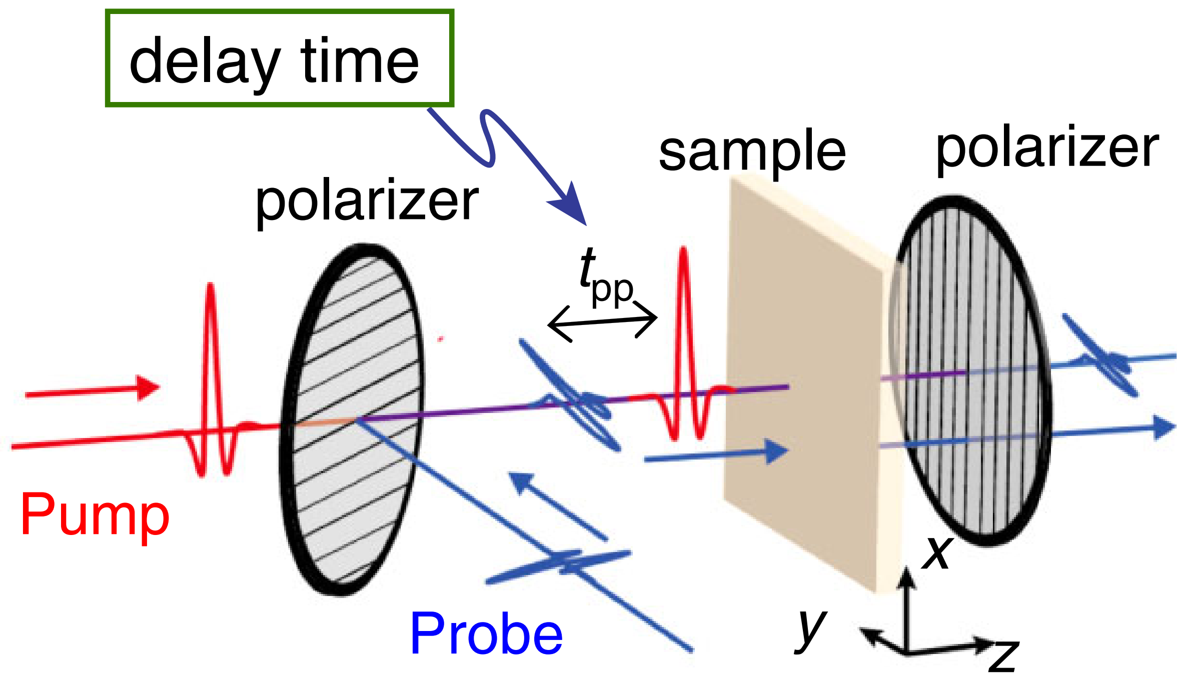
3. Results & Discussion

- Response of superconductor: Damped order parameter oscillations
- Generation of coherent phonons in non-equilibrium SC state:
  beating phenomena, resonant generation of coherent phonons
- Experimental signatures: Oscillations of pump-probe conductivity

4. Conclusions & Outlook

Motivation I: Recent pump-probe experiments

Matsunaga, Shimano, et al. PRL 111, 057002 (2013)



SC gap Δ:
$\sim 1 - 10$ meV

1 terahertz
≙ 4.1 meV
≙ 1 ps^{-1}
≙ 33 cm^{-1}

▶ THz-pump THz-probe spectroscopy:

- THz-pump pulse: Excite with intense femto-second pulse, induce dynamics
- THz-probe pulse: After delay time δt, measure with second, less intense pulse

▶ Optical conductivity / transmitivity as a function of ω and δt gives information about:

- Dynamics of SC condensate, order parameter oscillations
- Cooper pair recombination / recovery dynamics
- Coherent phonon oscillations

Motivation I: Recent pump-probe experiments

Matsunaga, Shimano, et al. PRL 109, 187002 (2012);PRL 111, 057002 (2013)

▶ THz-pump THz-probe spectroscopy on NbN films

$$|2\Delta| \simeq 3 \text{ meV}$$

- pump-pulse duration: 90 fs

→ non-adiabatic excitation of SC:

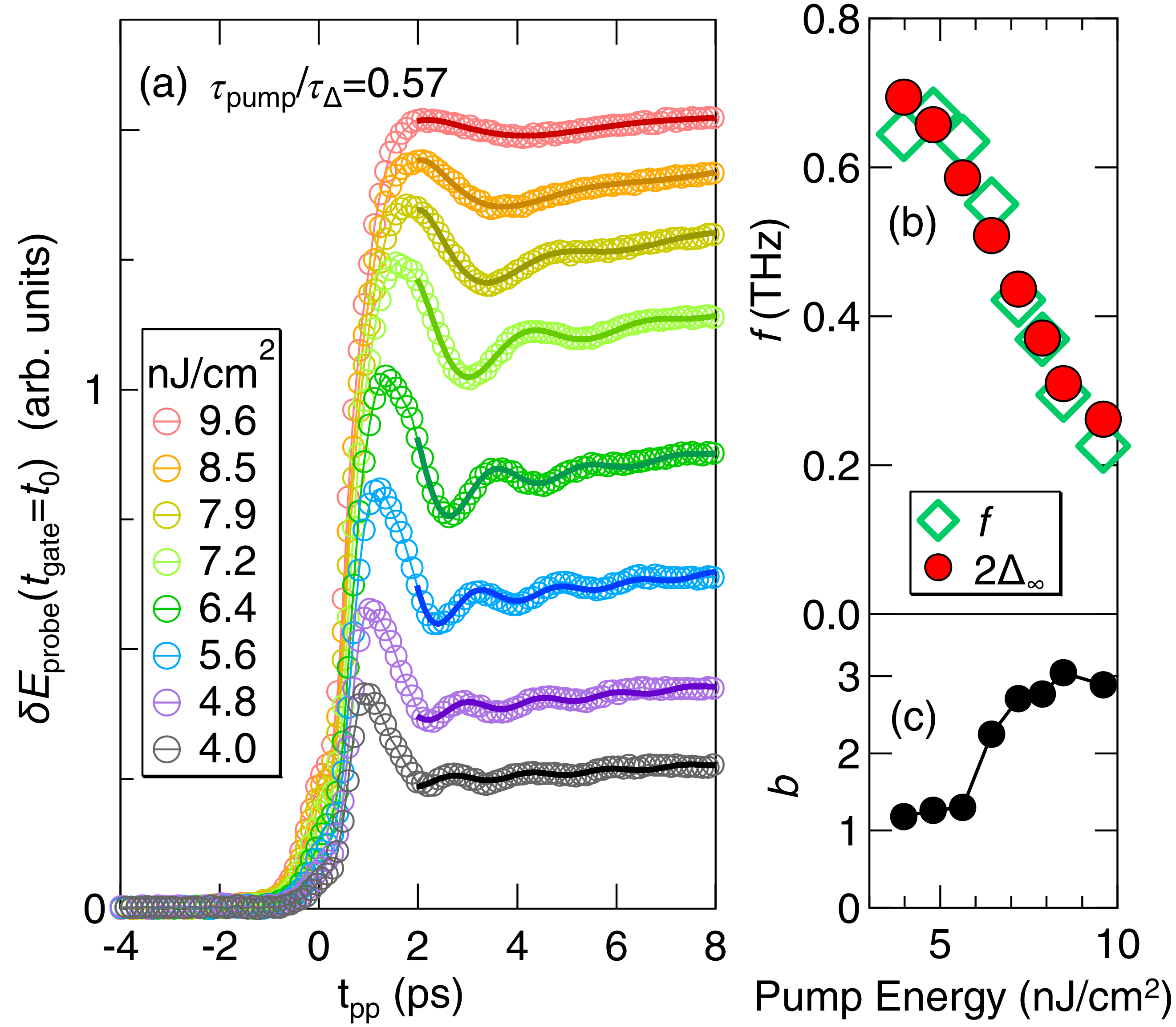
$$\tau_p \ll \tau_\Delta \sim h/(2|\Delta|)$$

- Measure change in transmission of probe field δE

▶ Observation:

- algebraically damped oscillations in δE as a function of delay time δt

- frequency changes with laser intensity



Interpretation:

→ order parameter amplitude oscillations

Motivation II: How does a quantum system relax?

? How does a quantum system thermalize?

As $t \to \infty$, generic observables $\hat{O}$ become time independent

- True relaxation vs. decoherence:

  -- Decoherence: Only averaged observables become time-independent

  - non-interacting systems, integrable systems *only* show decoherence

  -- True relaxation: Generic *local* (unaveraged) observables become time-independent

$$\bar{O} = \lim_{t\to\infty} \langle \Psi(t) | \, \hat{O} \, | \Psi(t) \rangle \ \text{ time independent}$$

  - (i) systems coupled to bath, (ii) closed systems with certain interactions (infinite closed systems; otherwise recurrence)

- Open questions:

  -- Which type of interactions lead to thermalization?

  -- Which observables $\hat{O}$ to consider?

  -- How to describe thermalized sate?
  (is there a density matrix (ensemble) ρ such that: $\bar{O} = \mathrm{Tr}[\hat{O}\,\rho]$?)

SC coupled to laser field and optical phonons

Goal: simulate *non-adiabatic* dynamics of superconductor coupled to
(i) pump laser field and (ii) optical phonons

- Microscopic model:

$$H = \sum_{\mathbf{k}s} \epsilon_{\mathbf{k}} c^{+}_{\mathbf{k}s} c_{\mathbf{k}s} + \sum_{\mathbf{k}} \left(\Delta_{\mathbf{k}} c^{+}_{\mathbf{k}\uparrow} c^{+}_{-\mathbf{k}\downarrow} + \mathrm{c.c} \right)$$

$$- \frac{e\hbar}{m} \sum_{\mathbf{kq}s} (\mathbf{k} \cdot \mathbf{A}_{\mathbf{q}}) c^{+}_{\mathbf{k}+\frac{\mathbf{q}}{2}s} c_{\mathbf{k}-\frac{\mathbf{q}}{2}s} + \frac{e^2}{2m} \sum_{\mathbf{kq}s} (\mathbf{A}_{\mathbf{q}-\mathbf{k}} \cdot \mathbf{A}_{\mathbf{q}}) c^{+}_{\mathbf{k}s} c_{\mathbf{k}s}$$

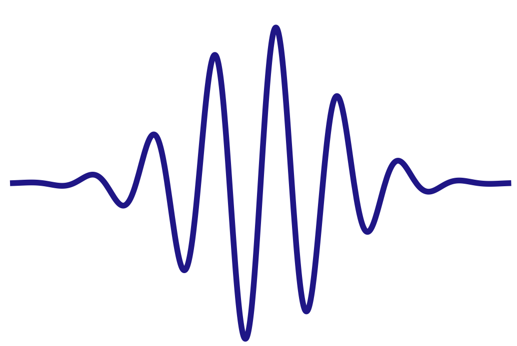
$$+ \sum_{\mathbf{q}j} \hbar\omega_{\mathbf{q}j} \left(b^{+}_{\mathbf{q}j} b_{\mathbf{q}j} + \frac{1}{2} \right) + \sum_{\mathbf{p}j\mathbf{k}s} \left(g_{\mathbf{pk}js} (b^{+}_{-\mathbf{p}j} + b_{\mathbf{p}j}) c^{+}_{\mathbf{k}+\mathbf{p},s} c_{\mathbf{k}s} + \mathrm{c.c.} \right)$$

with gap equation: $\Delta_{\mathbf{k}} = \sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \langle c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} \rangle$

- Gaussian pump and probe pulse: $\boldsymbol{A}_{\boldsymbol{q}}(t) \; = \; \boldsymbol{A}_0 \, e^{-(t/\tau)^2} \left(\delta_{\boldsymbol{q},\boldsymbol{q}_0} e^{-i\omega_0 t} + \delta_{\boldsymbol{q},-\boldsymbol{q}_0} e^{+i\omega_0 t} \right)$

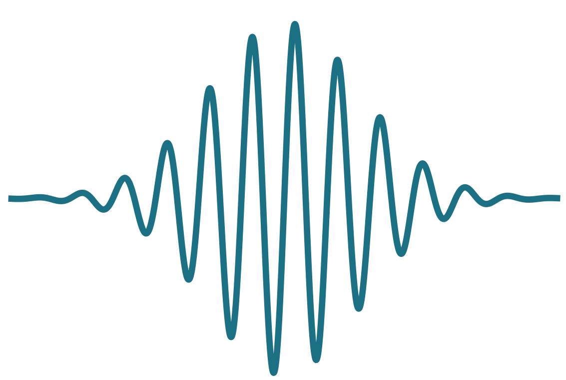
pump pulse:

$\hbar\omega_0 \gtrsim 2\,|\Delta|$



probe pulse:

$\hbar\omega_0 \lesssim 2\,|\Delta|$



Theoretical methods to compute non-equilibirum dynamics in SCs

Goal: simulate *non-adiabatic* dynamics of superconductor coupled to optical phonons

$\Longrightarrow$ considered hierarchies of time-scales: $\boxed{\tau_{\mathrm{p}} \ll \tau_{\mathrm{ph}}, \tau_{\Delta} \ll \tau_{\epsilon}}$

- *time-dependent Ginzburg Landau theory, mu*-, and T*- models:*
  - quasi-equilibrium is assumed at all times
  - time evolution of a single collective order parameter
  - only valid for: $\tau_{\Delta} \gg \tau_{\epsilon}$ (i.e., close to Tc, very dirty SCs)

- *Boltzmann kinetic equation:*
  - requires adiabaticity on all time-scales
  - does not capture coherent evolution of quasi-particle distributions
  - only valid for: $\tau_{\mathrm{p}} \gg \tau_{\Delta},\ \tau_{\mathrm{ph}}$

- *Phenomenological Rothwarf-Taylor models:*
  - rate equations for quasi-particle and phonon occupations
  - only valid for: $\tau_{\mathrm{p}} \gg \tau_{\Delta},\ \tau_{\mathrm{ph}}$

$\Longrightarrow$ instead use density matrix formalism (expansion in $\sim g_{\mathrm{ph}} t/\hbar$)

$$\frac{d}{dt}\mathcal{O} = \frac{i}{\hbar}\left[H, \mathcal{O}\right]$$

Density matrix formalism: Equations of motion

Superconducting state: Bogoliubov transformation

$$\alpha^\dagger_{\boldsymbol{k}} = u_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}\uparrow} + v^*_{\boldsymbol{k}} c_{-\boldsymbol{k}\downarrow} \qquad \beta^\dagger_{\boldsymbol{k}} = u_{\boldsymbol{k}} c^\dagger_{-\boldsymbol{k}\downarrow} - v^*_{\boldsymbol{k}} c_{\boldsymbol{k}\uparrow}$$

$$\Delta_{\boldsymbol{k}} = \sum_{\boldsymbol{k}} W_{\boldsymbol{k},\boldsymbol{k}'} \left[u_{\boldsymbol{k}'} v_{\boldsymbol{k}'} \left(1 - \left\langle \alpha^\dagger_{\boldsymbol{k}'} \alpha_{\boldsymbol{k}'} \right\rangle - \left\langle \beta^\dagger_{\boldsymbol{k}'} \beta_{\boldsymbol{k}'} \right\rangle \right) + u^2_{\boldsymbol{k}'} \left\langle \beta_{\boldsymbol{k}'} \alpha_{\boldsymbol{k}'} \right\rangle - v^2_{\boldsymbol{k}'} \left\langle \alpha^\dagger_{\boldsymbol{k}'} \beta^\dagger_{\boldsymbol{k}'} \right\rangle \right]$$

All quantities of interest can be expressed in terms of these dynamical variables

$$\left\langle \alpha^\dagger_{\boldsymbol{k}} \alpha_{\boldsymbol{k}'} \right\rangle (t), \quad \left\langle \beta^\dagger_{\boldsymbol{k}} \beta_{\boldsymbol{k}'} \right\rangle (t), \quad \left\langle \alpha^\dagger_{\boldsymbol{k}} \beta^\dagger_{\boldsymbol{k}'} \right\rangle (t), \quad \left\langle \alpha_{\boldsymbol{k}} \beta_{\boldsymbol{k}'} \right\rangle (t) \quad \left\langle b_{\mathbf{p}} \right\rangle (t), \quad \left\langle b^\dagger_{\mathbf{p}} \right\rangle (t)$$

- Current density:

$$\boldsymbol{j}(\boldsymbol{q},\omega) \simeq \frac{e\hbar}{mV} \sum_{\boldsymbol{k}} \boldsymbol{k} \Big[\left\langle \alpha^\dagger_{\boldsymbol{k}} \alpha_{\boldsymbol{k}+\boldsymbol{q}} \right\rangle - \left\langle \beta^\dagger_{\boldsymbol{k}+\boldsymbol{q}} \beta_{\boldsymbol{k}} \right\rangle + \boldsymbol{k} \cdot \boldsymbol{q} \frac{\hbar^2 \left| \Delta_1 \right|}{2m \left(E^2_{\boldsymbol{k}} \right)} \left(\left\langle \alpha^\dagger_{\boldsymbol{k}} \beta^\dagger_{\boldsymbol{k}+\boldsymbol{q}} \right\rangle + \left\langle \alpha_{\boldsymbol{k}+\boldsymbol{q}} \beta_{\boldsymbol{k}} \right\rangle \right) \Big]$$

- Lattice displacement: $U(\mathbf{r},t) = \sqrt{\dfrac{\hbar}{2M\omega_{\mathrm{ph}} V}} \sum_{\mathbf{p}} D_{\mathbf{p}}(t) e^{+i\mathbf{p}\cdot\mathbf{r}}$,

  with the *coherent phonon amplitude:* $D_{\mathbf{p}}(t) = \langle b_{\mathbf{p}} \rangle + \langle b^\dagger_{-\mathbf{p}} \rangle$

Density-matrix theory: $$\frac{d}{dt}\mathcal{O} = \frac{i}{\hbar} \left[H, \mathcal{O} \right]$$

$\Rightarrow$ yields equations of motions for the above expectation values

Density matrix formalism: Equations of motion

Density-matrix theory: $$\frac{d}{dt}\mathcal{O} = \frac{i}{\hbar}\left[H, \mathcal{O}\right]$$

$\Longrightarrow$ yields equations of motions for

$$\left\langle \alpha^\dagger_{\boldsymbol{k}} \alpha_{\boldsymbol{k}'} \right\rangle(t), \quad \left\langle \beta^\dagger_{\boldsymbol{k}} \beta_{\boldsymbol{k}'} \right\rangle(t), \quad \left\langle \alpha^\dagger_{\boldsymbol{k}} \beta^\dagger_{\boldsymbol{k}'} \right\rangle(t), \quad \left\langle \alpha_{\boldsymbol{k}} \beta_{\boldsymbol{k}'} \right\rangle(t) \quad \left\langle b_{\mathbf{p}} \right\rangle(t), \quad \left\langle b^\dagger_{\mathbf{p}} \right\rangle(t)$$

For example

$$\begin{aligned} i\hbar\frac{d}{dt}\left\langle \alpha^\dagger_{\boldsymbol{k}} \beta^\dagger_{\boldsymbol{k}+\boldsymbol{q}} \right\rangle = &-\left(R_{\boldsymbol{k}} + R_{\boldsymbol{k}+\boldsymbol{q}}\right)\langle a^\dagger_{\boldsymbol{k}} \beta^\dagger_{\boldsymbol{k}+\boldsymbol{q}}\rangle + C^*_{\boldsymbol{k}+\boldsymbol{q}}\langle \alpha^\dagger_{\boldsymbol{k}} \alpha_{\boldsymbol{k}+\boldsymbol{q}}\rangle + C^*_{\boldsymbol{k}}\left(\langle \beta^\dagger_{\boldsymbol{k}+\boldsymbol{q}} \beta_{\boldsymbol{k}}\rangle - \delta_{\boldsymbol{q},0}\right) \\ &+\frac{e\hbar}{2m}\sum_{\boldsymbol{q}'=\pm\boldsymbol{q}_0} 2\boldsymbol{k}\cdot\boldsymbol{A}_{\boldsymbol{q}'}(t)\Big\{-L^+_{\boldsymbol{k},\boldsymbol{q}'}\langle \alpha^\dagger_{\boldsymbol{k}+\boldsymbol{q}'} \beta^\dagger_{\boldsymbol{k}+\boldsymbol{q}}\rangle + L^+_{\boldsymbol{k}+\boldsymbol{q},-\boldsymbol{q}'}\langle \alpha^\dagger_{\boldsymbol{k}} \beta^\dagger_{\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{q}'}\rangle \\ &-M^-_{\boldsymbol{k}+\boldsymbol{q},-\boldsymbol{q}'}\langle \alpha^\dagger_{\boldsymbol{k}} \alpha_{\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{q}'}\rangle + M^-_{\boldsymbol{k},\boldsymbol{q}'}\left(\langle \beta^\dagger_{\boldsymbol{k}+\boldsymbol{q}} \beta_{\boldsymbol{k}+\boldsymbol{q}'}\rangle - \delta_{\boldsymbol{q},\boldsymbol{q}'}\right)\Big\} + \cdots \end{aligned}$$

Up to order $\left(\boldsymbol{A}_{\boldsymbol{q}}\right)^n$, laser field only couples $(\boldsymbol{k},\ \boldsymbol{k}')$ to $(\boldsymbol{k},\ \boldsymbol{k}' + n\boldsymbol{q})$

$\Longrightarrow$ leads to an effectively one-dimensional system of equations

$\Longrightarrow$ interaction with laser field can be computed essentially exactly.

[Papenkort, Kuhn, Axt, PRB 08]

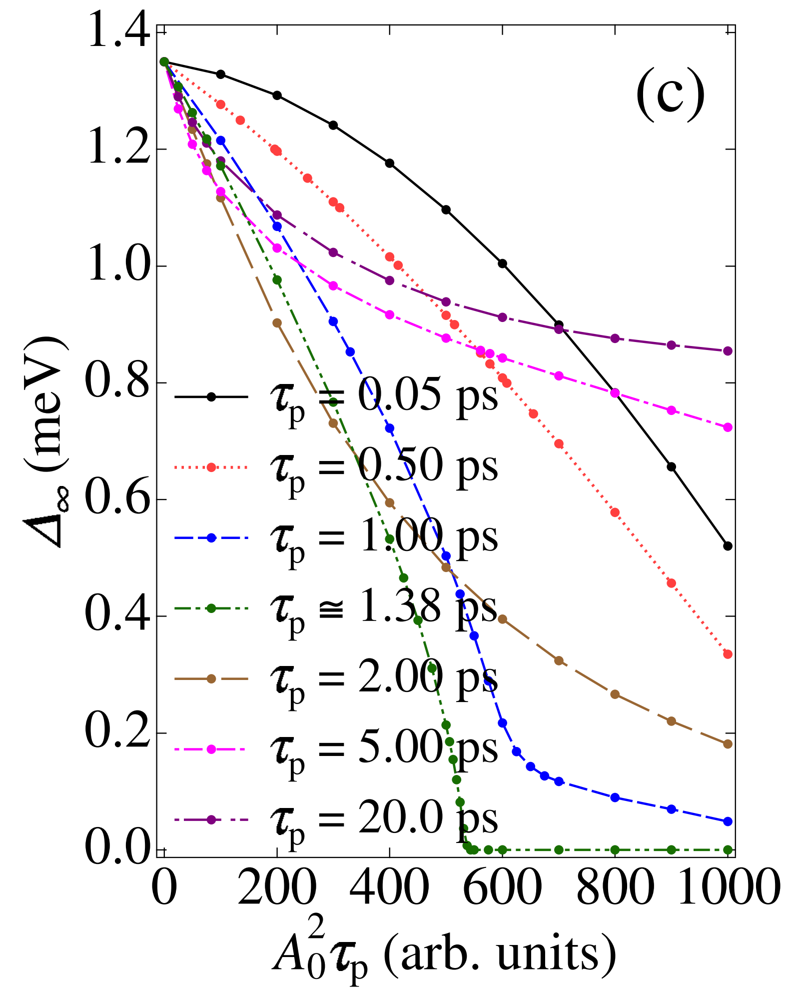
Response of superconductor (w/o phonons)

- Two regimes:
  - Adiabatic behavior for $\tau_\Delta \ll \tau_\mathrm{p}$
  - Non-adiabatic behavior for $\tau_\mathrm{p} \ll \tau_\Delta$
- Algebraically damped order parameter oscillations after short pump pulse ($\tau_\mathrm{p} \ll \tau_\Delta$):

$$|\Delta| = |\Delta_\infty| + \Gamma \frac{\cos(\frac{2|\Delta_\infty|}{\hbar}t + \Phi)}{\sqrt{t}}$$

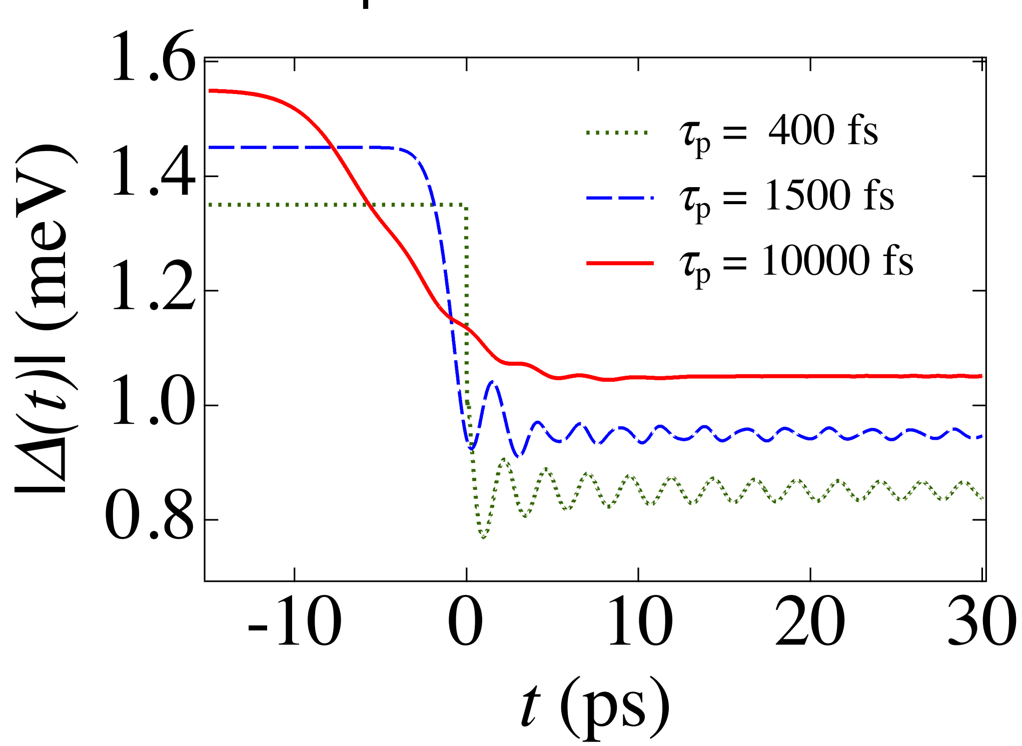
("Higgs" amplitude mode)

[Volkov, Kogan, JETP 74]
[Yuzbashyan, Altshuler PRL 06]

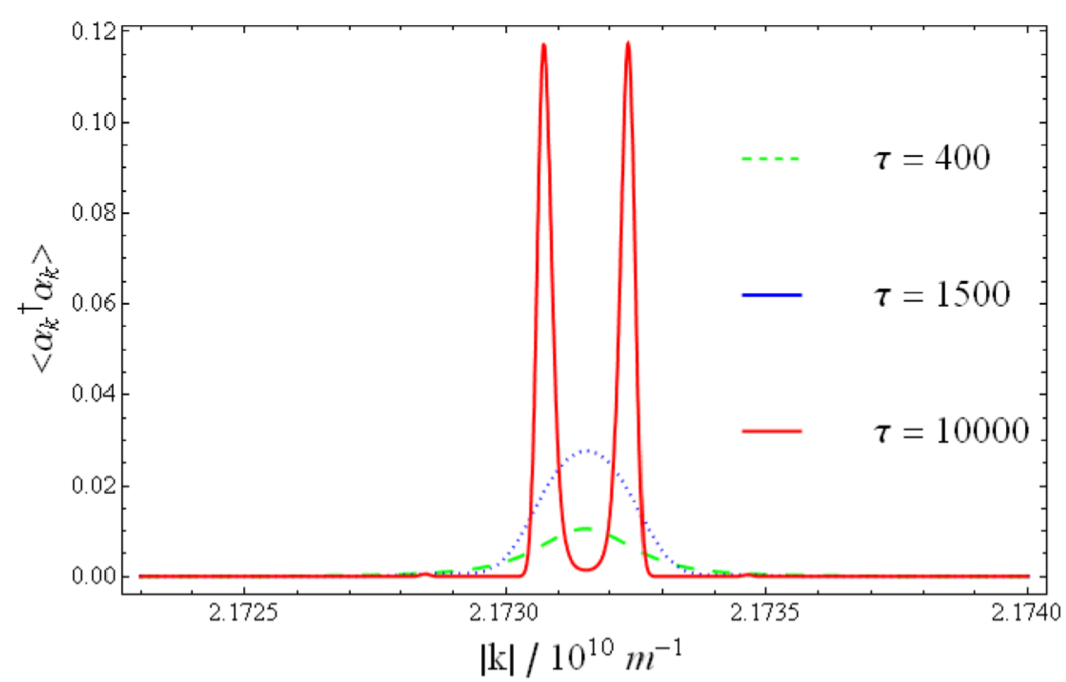


[APS, Manske, Avella, PRB 84, 214513 (2011)]

Order parameter oscillations



Quasiparticle occupations



Algebraically damped oscillations due to decoherence

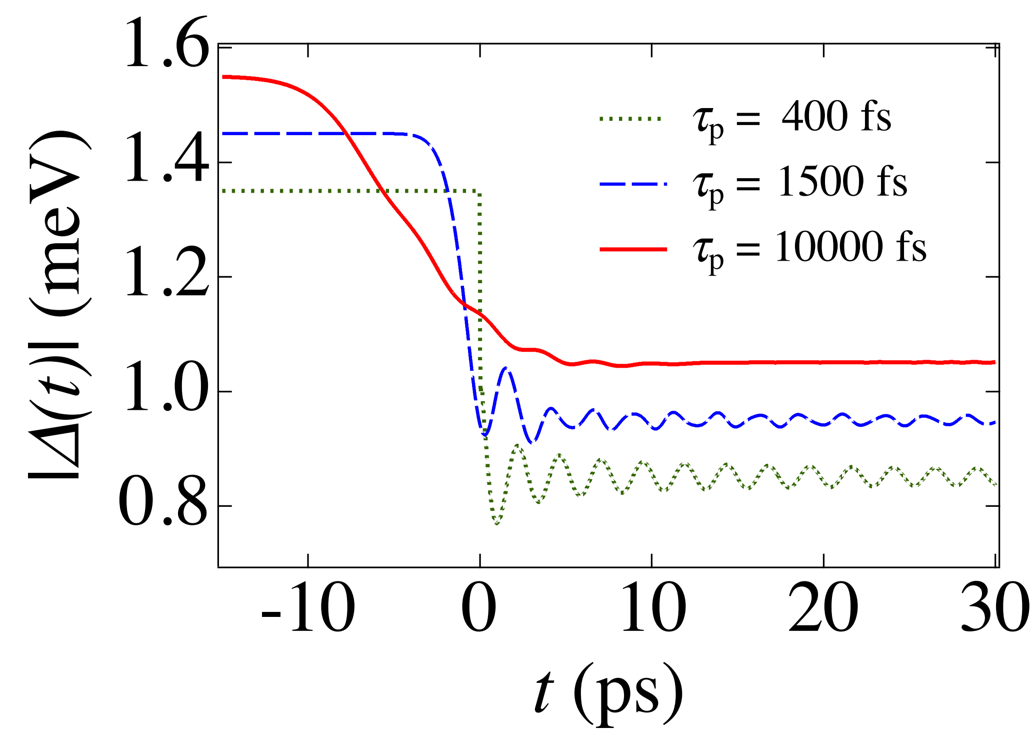
- Algebraically damped oscillations in averaged quantities due to decoherence

$$\Delta = W_0 \sum_{k \in \mathcal{W}} \langle c_{-k\downarrow} c_{+k\uparrow} \rangle$$

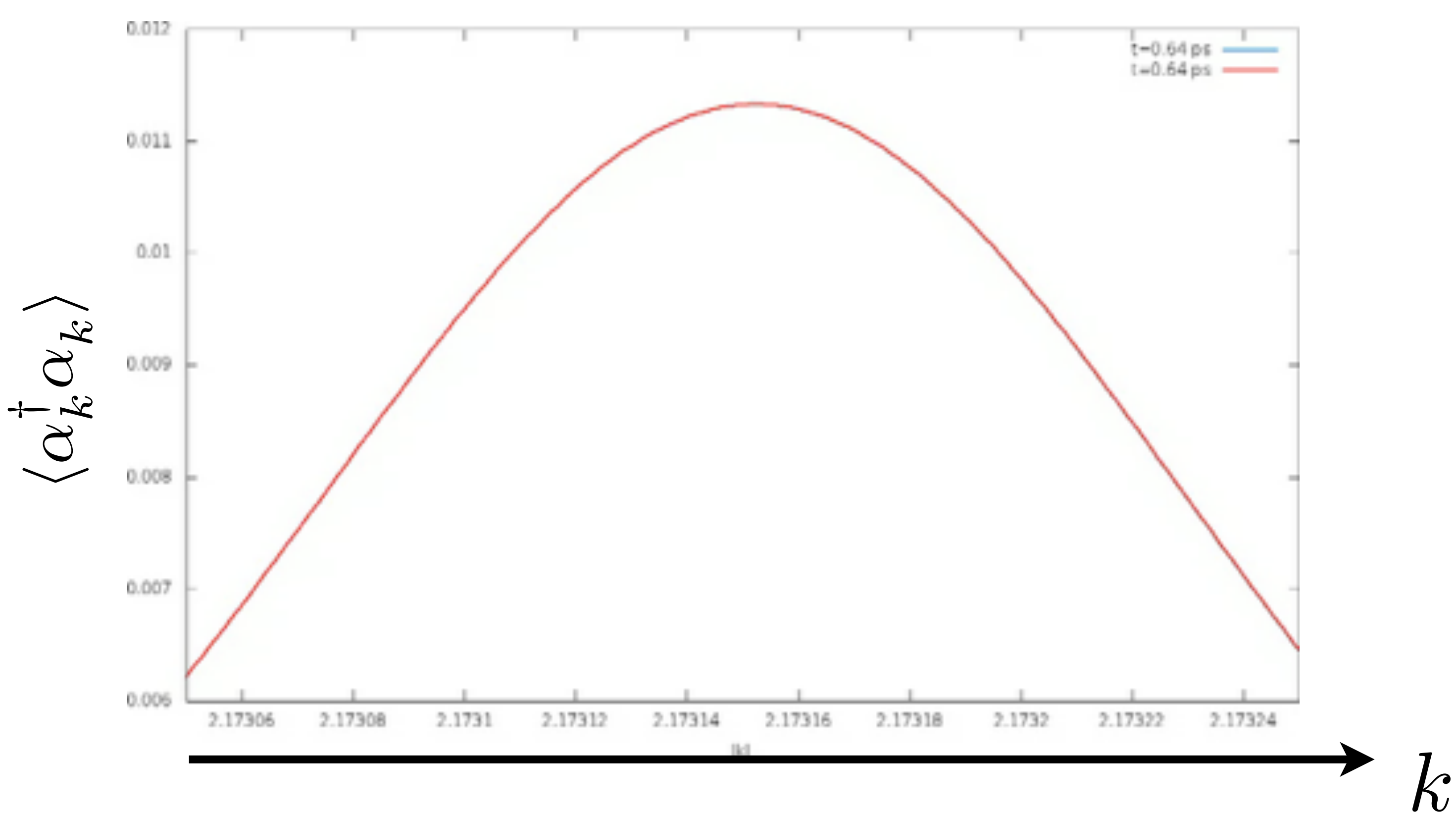
$$j_{q_0} = \frac{-e\hbar}{2mV} \sum_{k,\sigma} (2k + q_0) \langle c^\dagger_{k,\sigma} c_{k+q_0,\sigma} \rangle$$

- But no true relaxation!

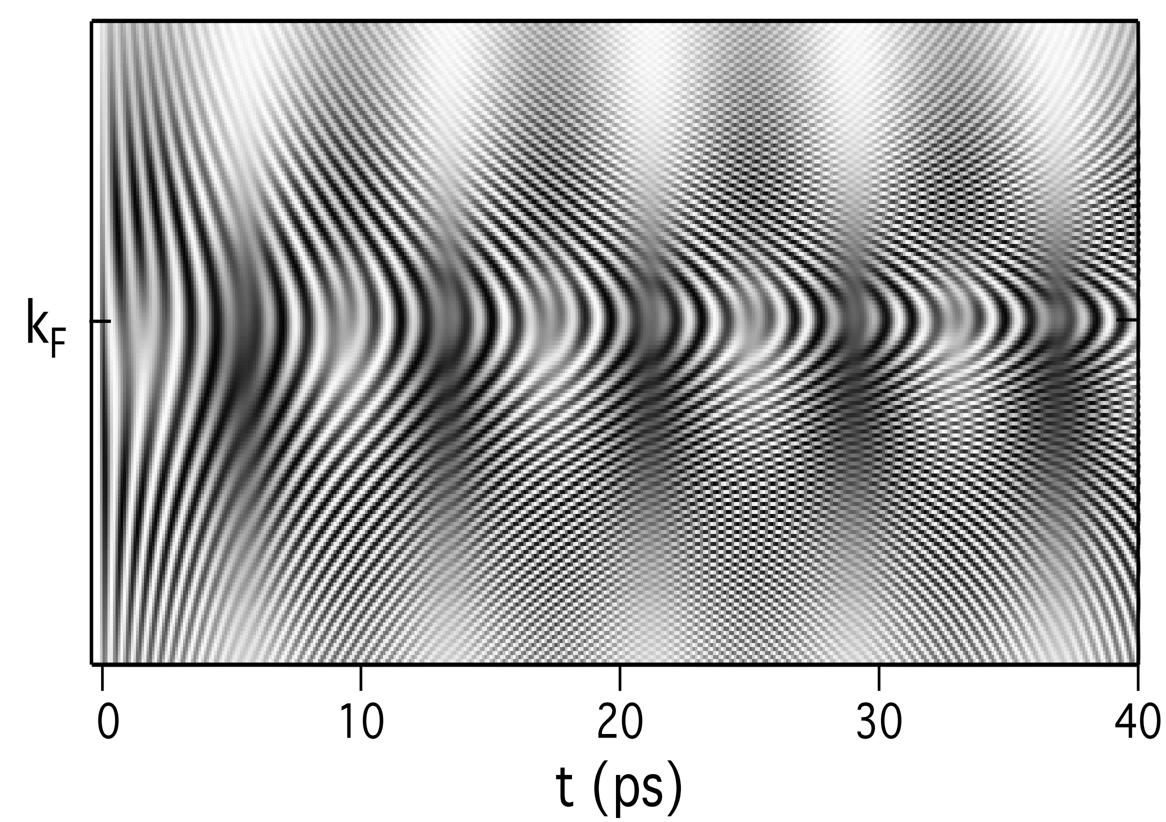
Order parameter oscillations



Quasiparticle occupations



Quasiparticle occupations $\langle \alpha^\dagger_k \alpha_k \rangle$

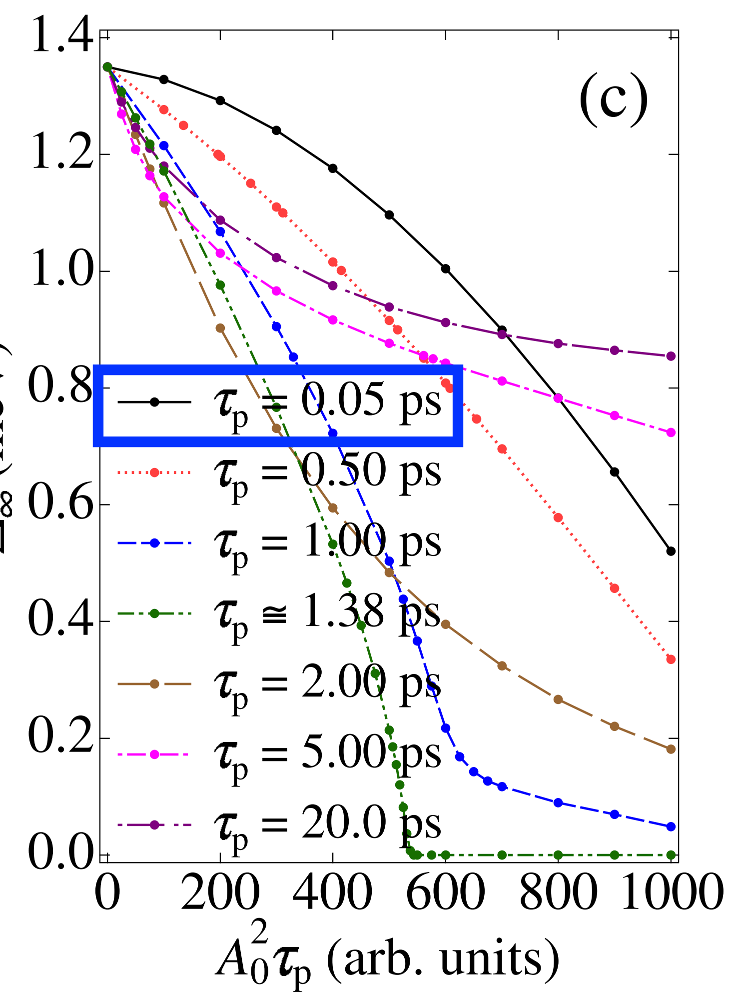


Gap oscillations: Comparison with experiment

- Qualitative agreement between theory and experiment

Numerical simulations:

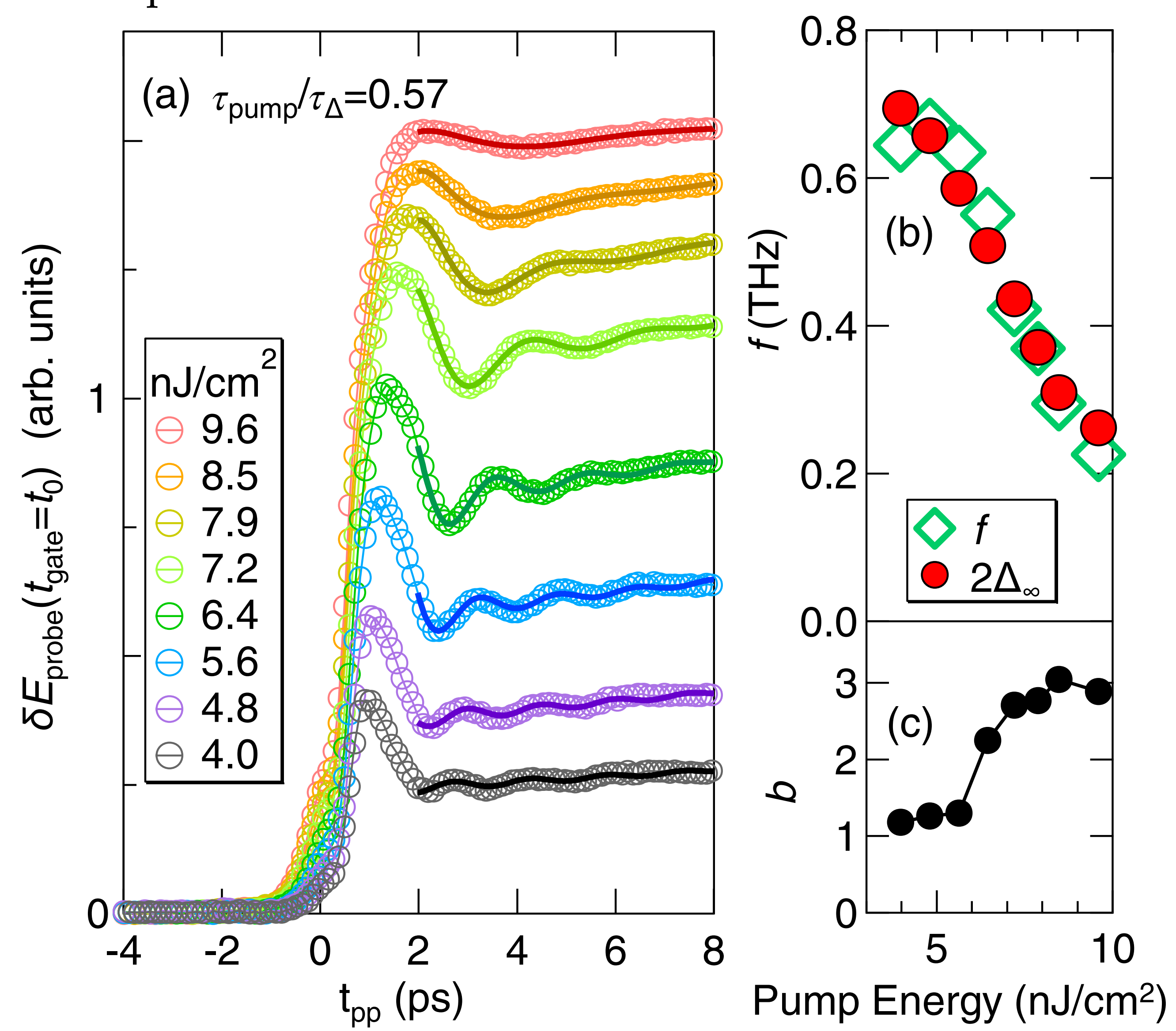
$$|\Delta| = |\Delta_\infty| + \Gamma \frac{\cos(\frac{2|\Delta_\infty|}{\hbar}t + \Phi)}{\sqrt{t}}$$



APS. Manske, Avella, PRB 84, 214513 (2011)

THz-pump THz-probe spectroscopy NbN:

$\tau_p = 60$ fs



Matsunaga, Shimano, et al.
PRL 111, 057002 (2013)

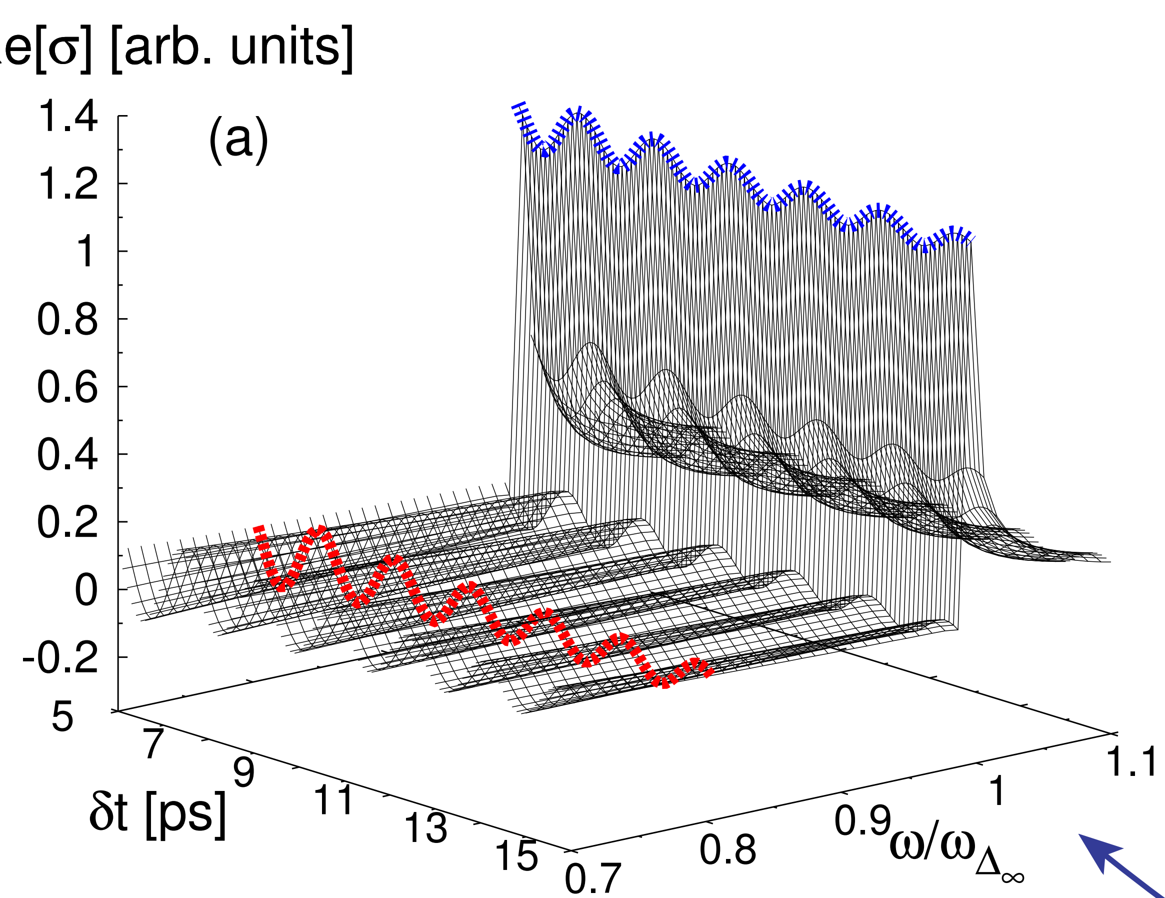
Response of superconductor (w/o phonons)

- Pump-probe conductivity shows signatures of non-adiabatic dynamics

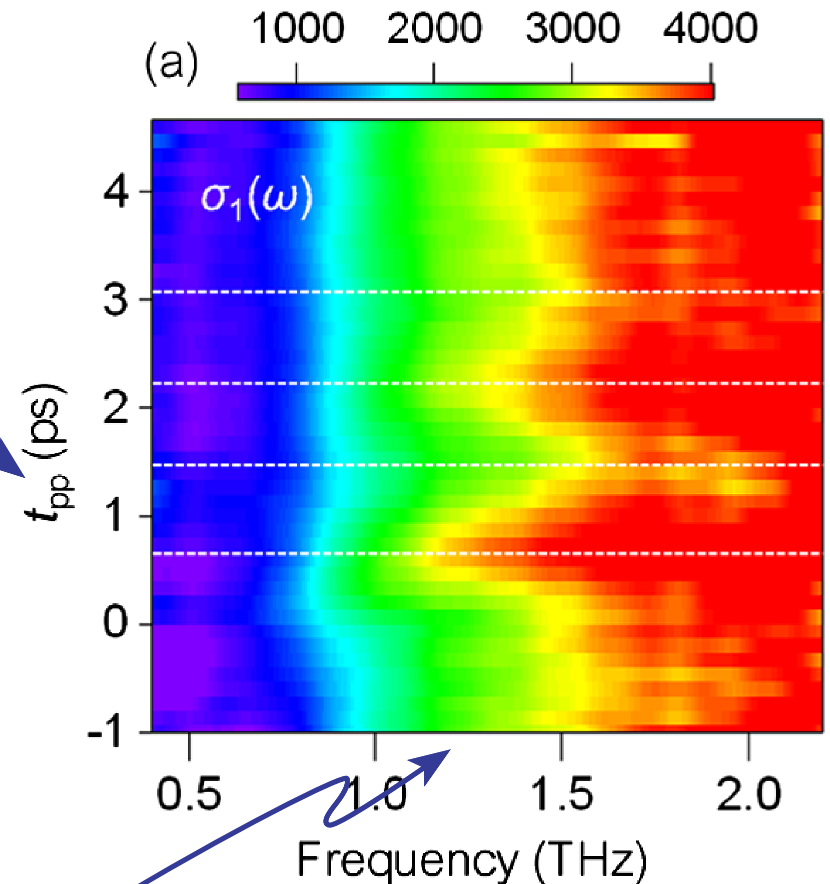
$$\sigma(\delta t, \omega) = j(\omega)/[i\omega A(\omega)]$$

$\Rightarrow$ oscillations in pump-probe response as a function of delay time δt

Numerical simulation:



THz-pump THz-probe spectroscopy NbN:



delay time

$2|\Delta|$

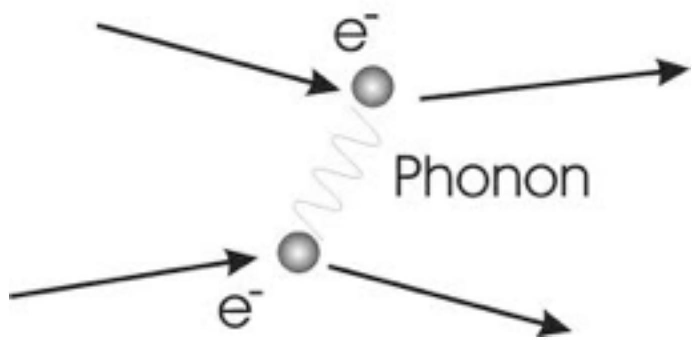
arXiv:1309.7318 (2013)

Matsunaga, Shimano, et al. PRL 111, 057002 (2013)

Superconductor coupled to optical phonons

▶ Linear coupling to optical phonons:

$$H_{\text{el-ph}} = g_{\text{ph}} \sum_{\mathbf{p},\mathbf{k},\sigma} \left[(b^\dagger_{-\mathbf{p}} + b_{\mathbf{p}}) c^\dagger_{\mathbf{k}+\mathbf{p},\sigma} c_{\mathbf{k},\sigma} + \text{c.c.} \right]$$



- perform expansion in g_{ph} using $\frac{d}{dt}\mathcal{O} = \frac{i}{\hbar}[H, \mathcal{O}]$

$\Rightarrow$ infinite hierarchy of equations of motion

▶ Study generation of coherent phonons:

- Break hierarchy at first order: $\langle \alpha^\dagger_{\mathbf{k}} \beta^\dagger_{\mathbf{k}'} b_{\mathbf{p}} \rangle \simeq \langle \alpha^\dagger_{\mathbf{k}} \beta^\dagger_{\mathbf{k}'} \rangle \langle b_{\mathbf{p}} \rangle$

- Non-vanishing $\langle b_{\mathbf{p}} \rangle$ leads to finite lattice displacement:

$$U(\mathbf{r}, t) = \sqrt{\frac{\hbar}{2M\omega_{\text{ph}} V}} \sum_{\mathbf{p}} D_{\mathbf{p}}(t) e^{+i\mathbf{p}\cdot\mathbf{r}},$$

with $D_{\mathbf{p}} = (b^\dagger_{-\mathbf{p}} + b_{\mathbf{p}})$

- Equation of motion for coherent phonon amplitude:

forced harmonic oscillator

$$\boxed{\left[\frac{d^2}{dt^2} + \omega^2_{\text{ph}}\right] D_{\mathbf{p}}(t) = \mathcal{F}_{\mathbf{p}}(t),}$$

with forcing term:

$$\mathcal{F}_{\mathbf{p}}(t) = -\frac{2\omega_{\text{ph}}}{\hbar} g_{\text{ph}} \sum_{\mathbf{k}} \left[M^+_{\mathbf{k},\mathbf{p}} \left(\langle \alpha_{\mathbf{k}+\mathbf{p}} \beta_{\mathbf{k}} \rangle - \langle \alpha^\dagger_{\mathbf{k}} \beta^\dagger_{\mathbf{k}+\mathbf{p}} \rangle \right) + L^-_{\mathbf{k},\mathbf{p}} \left(\langle \alpha^\dagger_{\mathbf{k}} \alpha_{\mathbf{k}+\mathbf{p}} \rangle + \langle \beta^\dagger_{\mathbf{k}+\mathbf{p}} \beta_{\mathbf{k}} \rangle \right) \right],$$

Equation of motion for coherent phonon amplitude

- Equation of motion for coherent phonon amplitude:

forced harmonic oscillator

$$\left[\frac{d^2}{dt^2}+\omega_{\rm ph}^2\right]D_{\mathbf{p}}(t)=\mathcal{F}_{\mathbf{p}}(t), \qquad \mathcal{F}_{\mathbf{p}}(t)=-\frac{2\omega_{\rm ph}}{\hbar}g_{\rm ph}\sum_k\left[M^{+}_{k,p}\left(\langle\alpha_{k+p}\beta_k\rangle-\langle\alpha_k^\dagger\beta_{k+p}^\dagger\rangle\right)+\cdots\right]$$

▶ Hierarchy of time scales: $\boxed{\tau_{\rm p}\ll\tau_\Delta\ll\tau_{\rm ph}}$

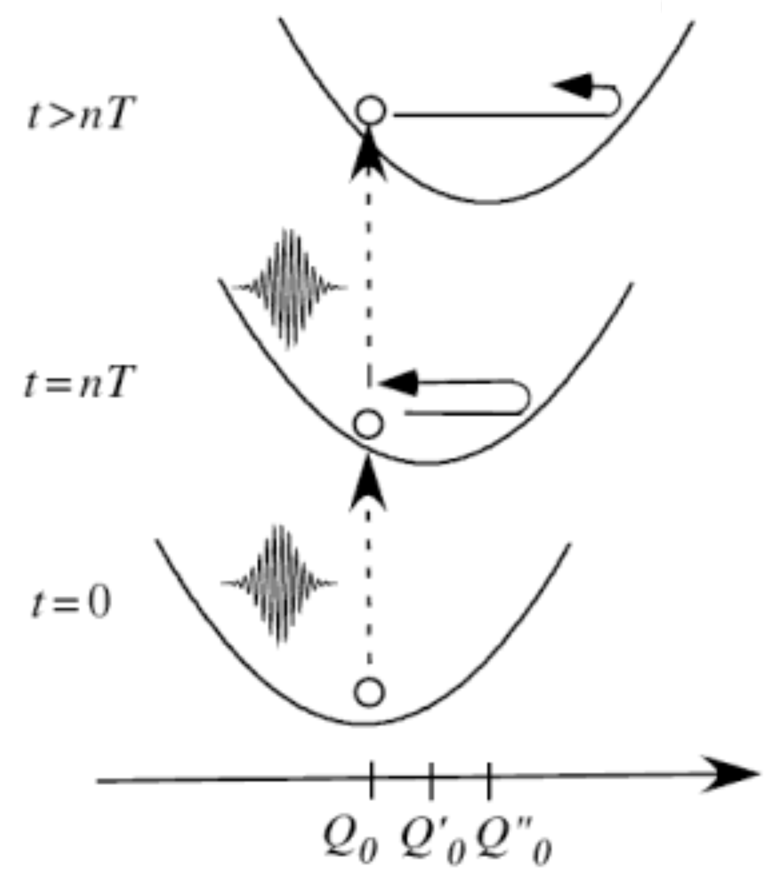
step-function

$\Rightarrow$ Forcing term can be approximated by $\mathcal{F}_{\mathbf{p}}(t)\simeq A_{\mathbf{p}}\Theta(t)$

$$\Rightarrow\ D_{\mathbf{p}}(t)\simeq\frac{A_{\mathbf{p}}}{\omega_{\rm ph}^2}[1-\cos(\omega_{\rm ph}t)]$$

▶ Displacive excitation of coherent phonons:
(similar to semiconductors)

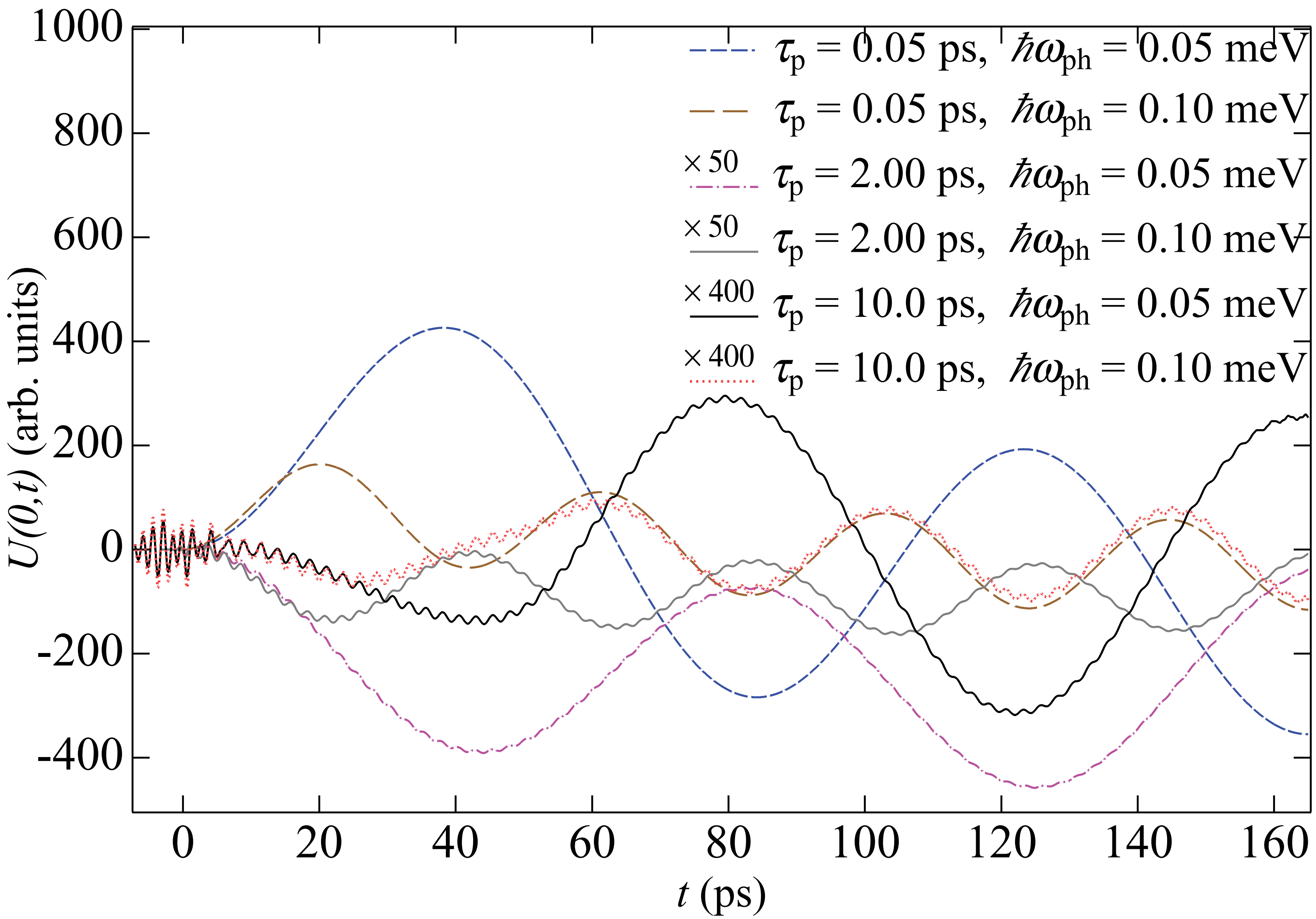
- abrupt change in quasiparticle states leads to jump in equilibrium position of lattice
- cosine oscillations in lattice displacement
- extrema at integer and half-integer values of $\tau_{\rm ph}$



Generation of coherent phonons: Numerical results

Hierarchy of time scales: $\boxed{\tau_{\rm p} \ll \tau_\Delta \ll \tau_{\rm ph}}$ $\Rightarrow$ $D_{\mathbf{p}}(t) \simeq \frac{A_{\mathbf{p}}}{\omega_{\rm ph}^2}[1 - \cos(\omega_{\rm ph} t)]$

- ▶ Numerical results for displacive excitation of coherent phonons:



- cosine oscillations in lattice displacement $U(R_0, t)$

- extrema at integer and half-integer values of $\tau_{\rm ph}$

APS, Manske, Avella, PRB 84, 214513 (2011)

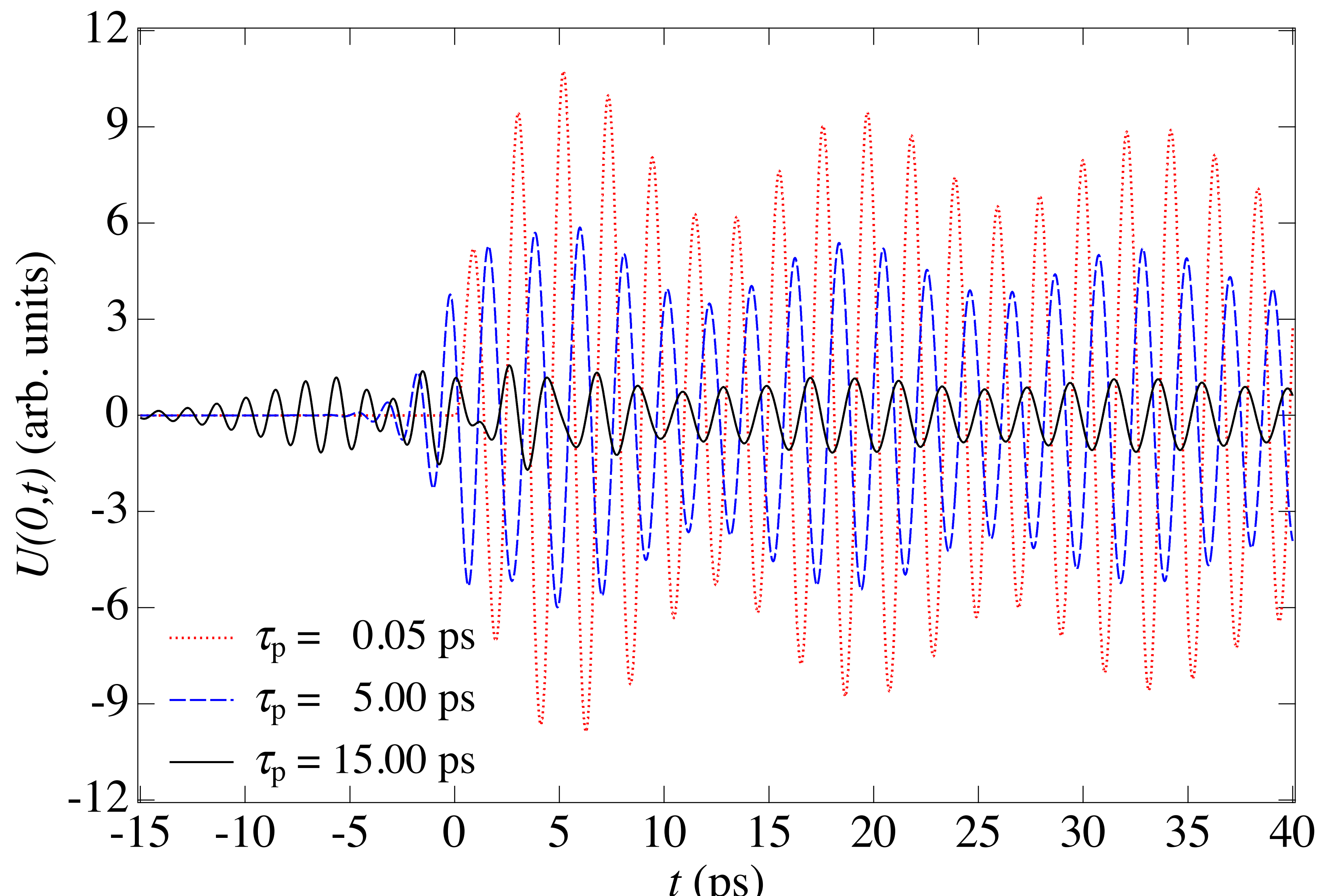
Generation of coherent phonons: Quantum beats

Hierarchy of time scales: $\boxed{\tau_{\mathrm{p}} \ll \tau_{\Delta} \sim \tau_{\mathrm{ph}}}$

$\Rightarrow$ forcing term can be approximated by $\mathcal{F}_{\mathbf{p}}(t) \simeq \Theta(t)[A_{\mathbf{p}} + B_{\mathbf{p}} \cos\left(2\Delta_{\infty} t/\hbar\right)/\sqrt{t}]$,

$\Rightarrow$ quantum beating when $\omega_{\mathrm{d}} = \left|\omega_{\Delta_{\infty}} - \omega_{\mathrm{ph}}\right| \ll \omega_{\mathrm{ph}}$

$$\Rightarrow \quad D_{\mathbf{p}}(t) \simeq \frac{B_{\mathbf{p}}}{\omega_{\mathrm{ph}}}\sqrt{\frac{\pi}{2\omega_{\mathrm{d}}}}[\cos(t\omega_{\mathrm{ph}})S_2(t\omega_{\mathrm{d}}) + \sin(t\omega_{\mathrm{ph}})C_2(t\omega_{\mathrm{d}})],$$

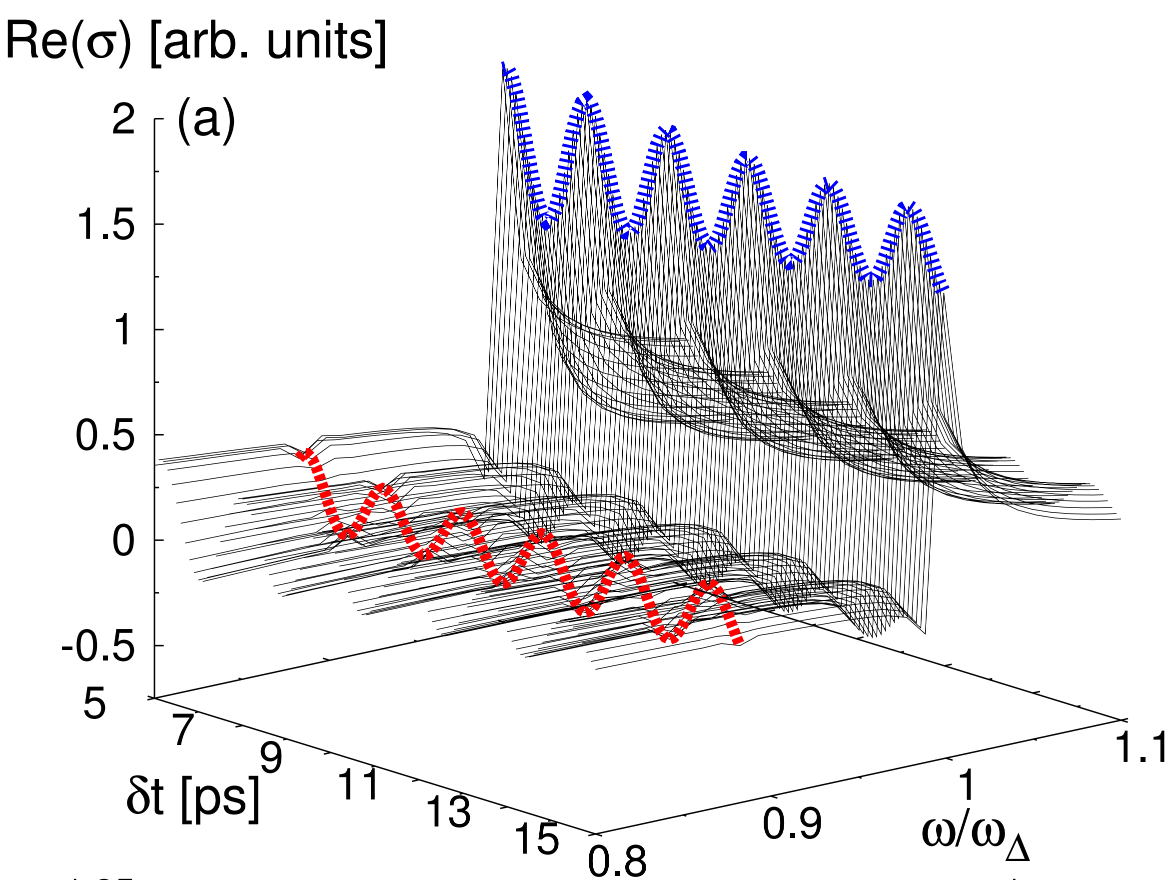


APS, Manske, Avella, PRB 84, 214513 (2011)

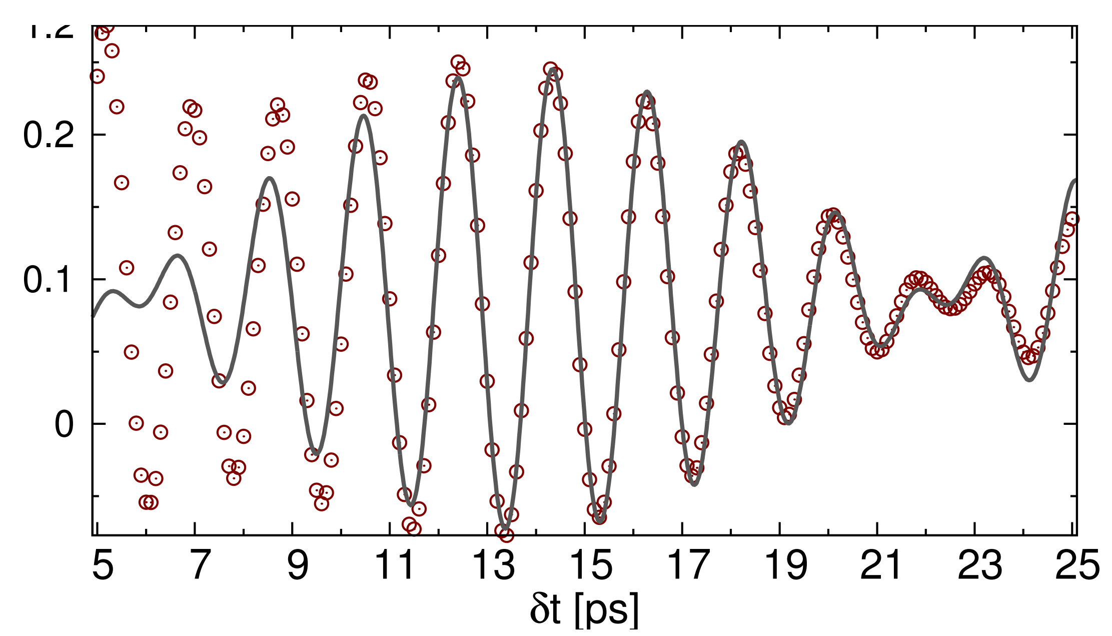
Generation of coherent phonons: Pump-probe conductivity

Hierarchy of time scales: $\boxed{\tau_{\mathrm{p}} \ll \tau_{\Delta} \sim \tau_{\mathrm{ph}}}$

▶ Signatures of non-adiabatic dynamics in pump-probe conductivity



$\mathrm{Re}[\sigma(\delta t, \omega = \omega_{\mathrm{ph}})]$



$\Longrightarrow$ beating phenomenon in pump-probe conductivity $\sigma(\delta t, \omega = \omega_{\mathrm{ph}})$ as a function of delay time δt

arXiv:1309.7318 (2013)

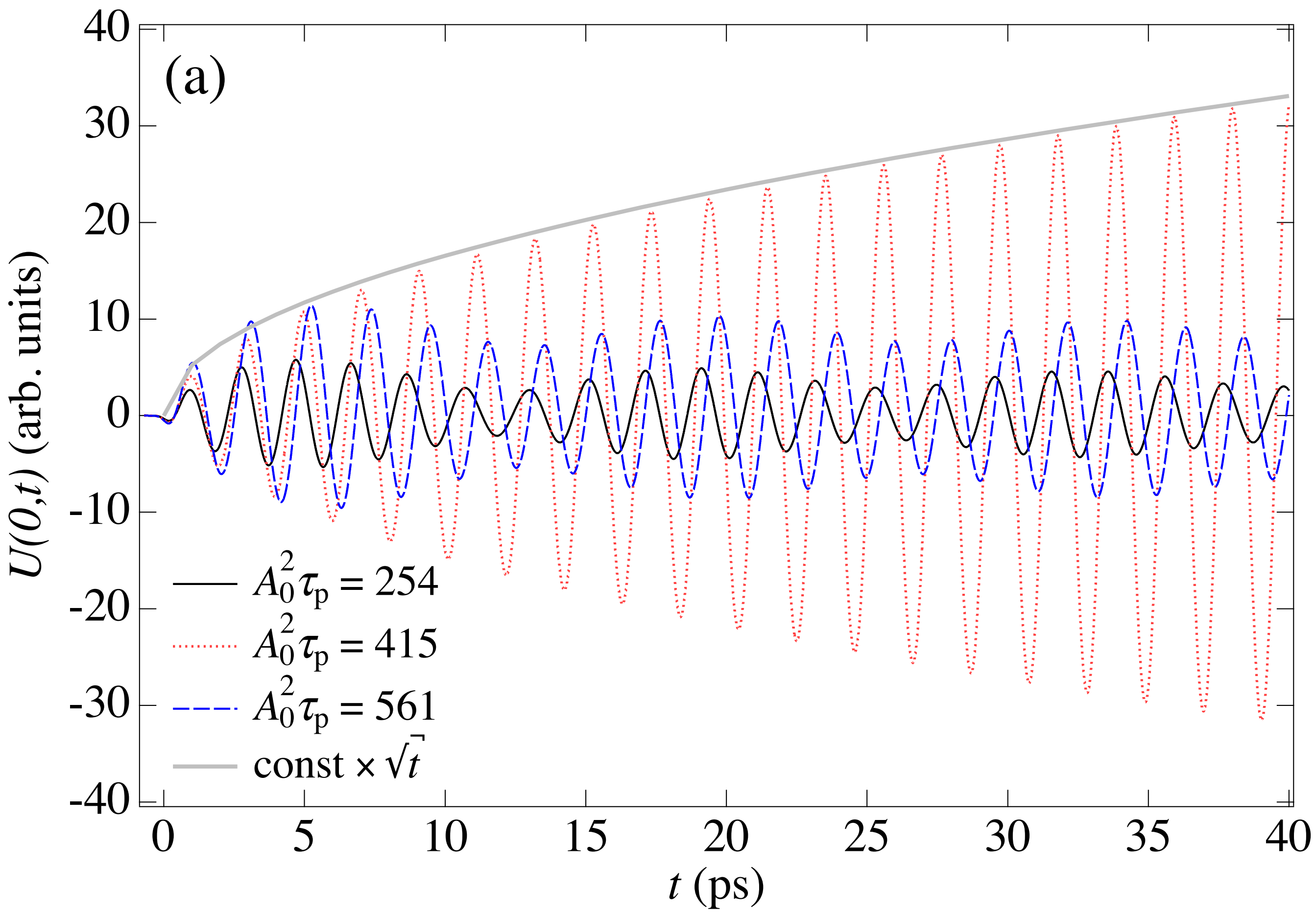
Generation of coherent phonons: Resonance

Hierarchy of time scales: $\boxed{\tau_{\mathrm{p}} \ll \tau_{\Delta} = \tau_{\mathrm{ph}}}$

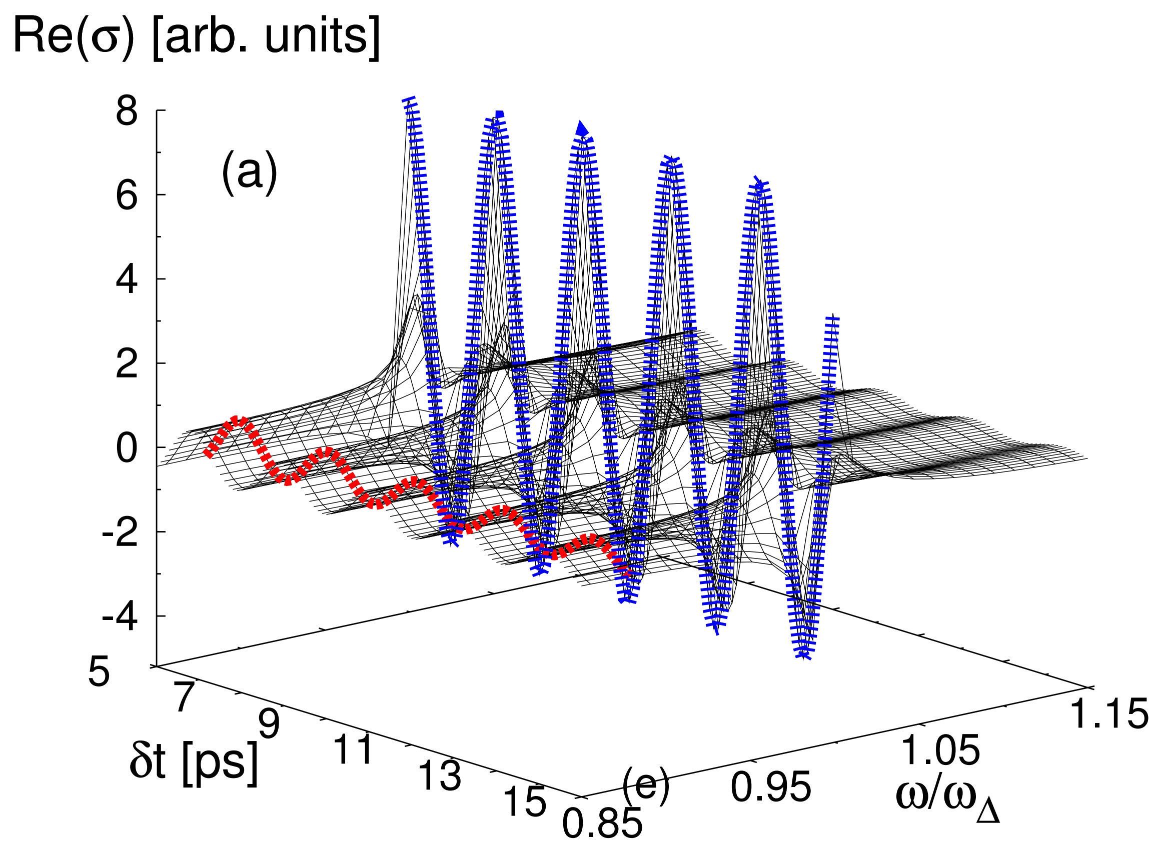
$\Rightarrow$ resonant generation of coherent phonons for $\omega_{\mathrm{ph}} = \omega_{\Delta_\infty} (= 2\Delta_\infty/\hbar)$

$$\Rightarrow \quad D_{\mathbf{p}}(t) \simeq \frac{B_{\mathbf{p}}}{\omega_{\mathrm{ph}}} \sqrt{t} \sin(\omega_{\mathrm{ph}} t) + \cdots$$

Lattice displacement:



Pump-probe conductivity:



$\Longrightarrow$ resonantly enhanced oscillations in pump-probe conductivity as a function of delay time δt

arXiv:1309.7318 (2013)

Conclusions & Outlook

▶ Microscopic simulation of ultrafast dynamics in superconductors

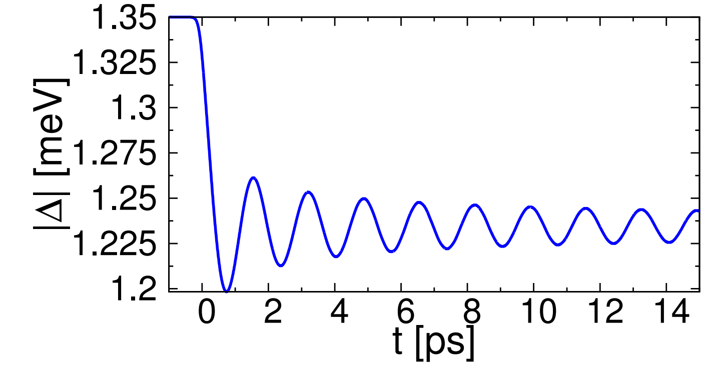
▶ Non-adiabatic regime $\tau_{\rm p} \ll \tau_\Delta, \tau_{\rm ph}$:

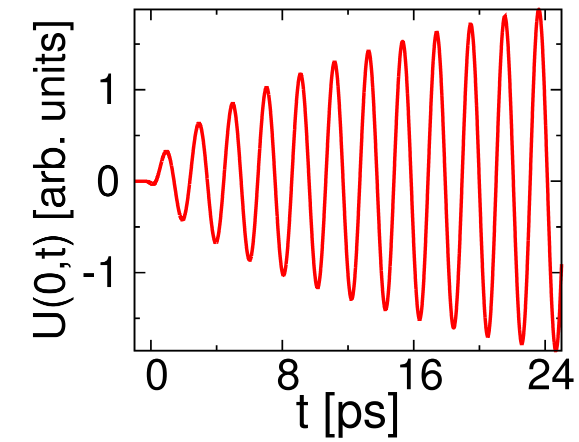
- order parameter oscillations

  $\Rightarrow$ qualitative agreement with experiment

- generation of coherent phonos

- for $\hbar\omega_{\rm ph} = 2\Delta_\infty$: resonant enhancement of coherent phonons

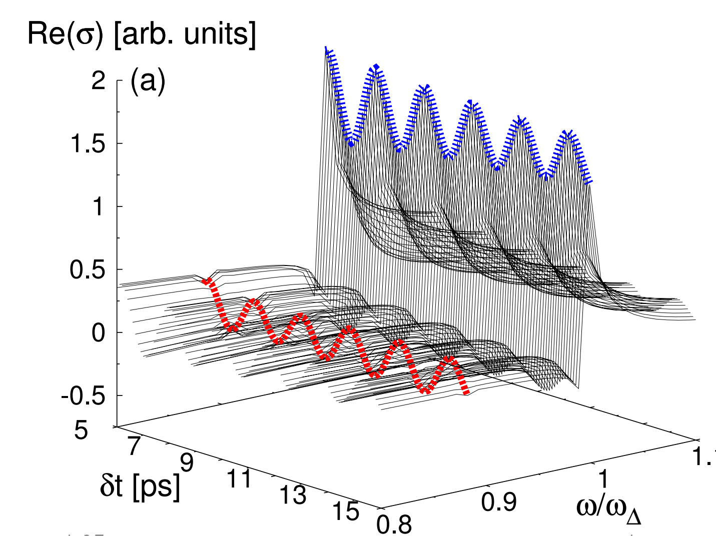




▶ Pump-probe conductivity:

- oscillations in $\sigma(\delta t, \omega)$ as a function of delay time with frequencies $\omega_{\Delta_\infty} = 2\Delta_\infty/\hbar$ and $\omega_{\rm ph}$

- strong enhancement of oscillation amplitude when frequencies are in resonance: $\omega_{\Delta_\infty} = \omega_{\rm ph}$



▶ **Outlook:**

- consider higher order in correlation expansion:

  incoherent phonons, feedback on SC condensate

PRB 84, 214513 (2011); EPL 101, 17002 (2012); arXiv:1309.7318 (2013)