

The Probes (perturbations)

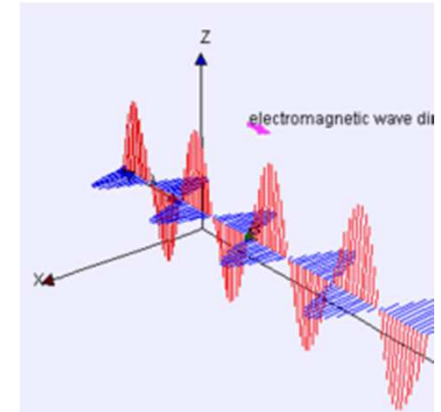
1) Electro-magnetic field

$$\vec{E} = \hat{e} E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\vec{B} = \frac{1}{\omega} \vec{k} \times \vec{E}$$

$$E = \hbar\omega = \frac{hc}{\lambda}$$

$$E(\lambda = 1\text{\AA}) = 12.4 \text{ keV}$$



2) Particles

$$\psi \sim e^{i\vec{k} \cdot \vec{r}}$$

Particle	electron	positrons	muons	neutrons	proton	Nuclei ${}^A_Z X$	ions ${}^A_Z X^{n\pm}$
Mass	$m_e = 9,1 \times 10^{-31} \text{ kg}$	$m_p = 9,1 \times 10^{-31} \text{ kg}$	$m_\mu = 1,88 \times 10^{-28} \text{ kg}$	$M_N = 1,675 \times 10^{-27} \text{ kg}$	$M_p = 1,673 \times 10^{-27} \text{ kg}$	$Z \cdot M_p + (A-Z) \cdot M_N - E_B/c^2$	$\sim Z \cdot M_p + (A-Z) \cdot M_N - E_B/c^2$
Charge	-e	+e	-e	0	+e	+Ze	$\pm ne$
Spin	1/2	1/2	1/2	1/2	1/2	0 if Z and A are even $S > 1/2$ for 75% of isotopes	Depends on n ($\mu_e \gg \mu_{\text{Nucleus}}$)

(Internal probe)

neutron

mass

$$m_n = 1.675 \times 10^{-27} \text{ kg}$$

charge

$$0$$

spin

$$s = 1/2$$

magnetic dipole moment

$$\mu_n = \frac{-e\hbar}{2m_n} g s_n \quad \text{with } g_n = 3.826$$

energy

$$E = \frac{\hbar^2 k^2}{2m_n} \quad k = \frac{2\pi}{\lambda}$$

$$E [\text{meV}] = \frac{81.81}{\lambda^2 [\text{\AA}]}$$

interaction with matter:

Coulomb interaction

—

strong-force interaction

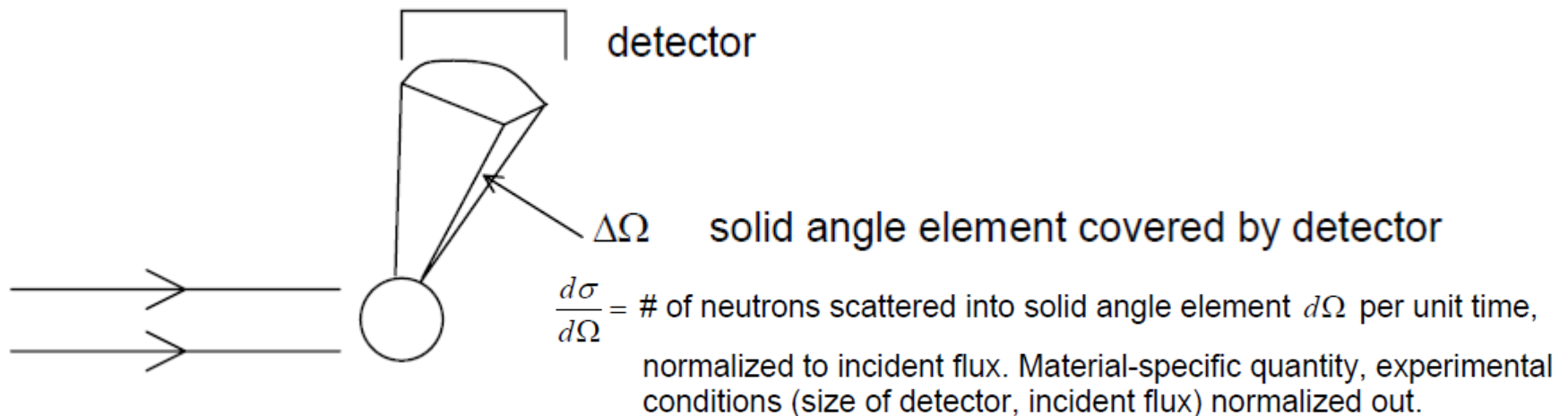
✓

magnetic dipole-dipole
interaction✓

=> For wave-lengths \sim interatomic distances, $E_{\text{Neutron}} \sim 10 \text{ meV}$

Interaction of Neutrons with Matter

	elastic scattering	inelastic scattering
strong-force interaction ("nuclear scattering")	position of nuclei in solid (lattice structure)	lattice vibrations (phonons)
magnetic interaction	position and orientation of electronic magnetic moments in solids (ferromagnetism, antiferromagnetism)	spin excitations (magnons, spin waves)



dimensions: $\left[\frac{d\sigma}{d\Omega} \right] = \frac{1}{[\Delta\Omega][t][\phi]} = \text{area}$

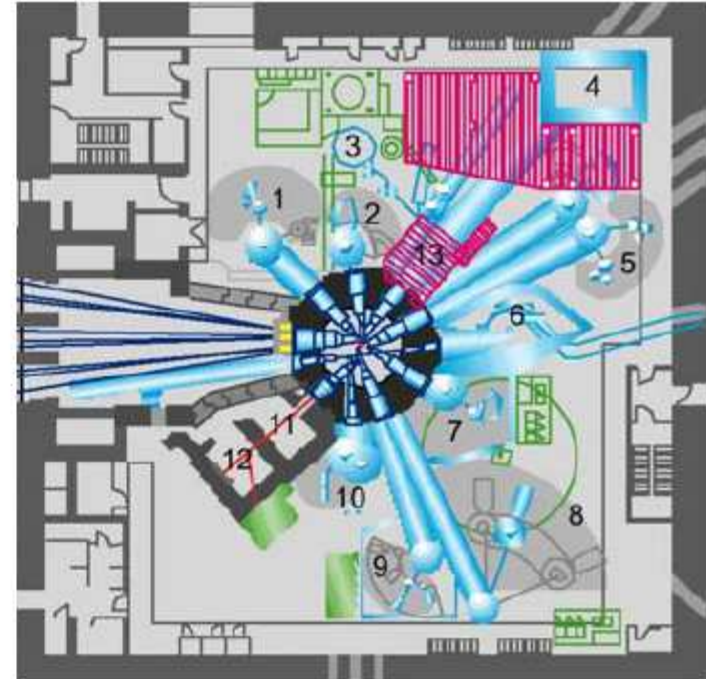
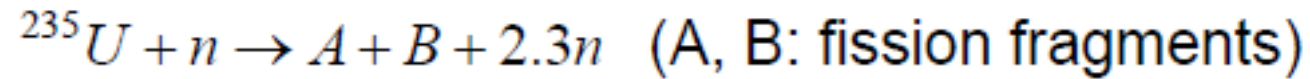
↑
dimensionless

Neutron Sources Worldwide



- Research reactors
- Spallation sources

Research Reactors

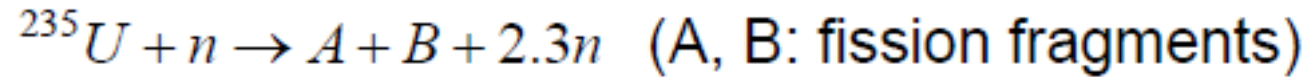


Research reactor FRM-II in Munich, Germany

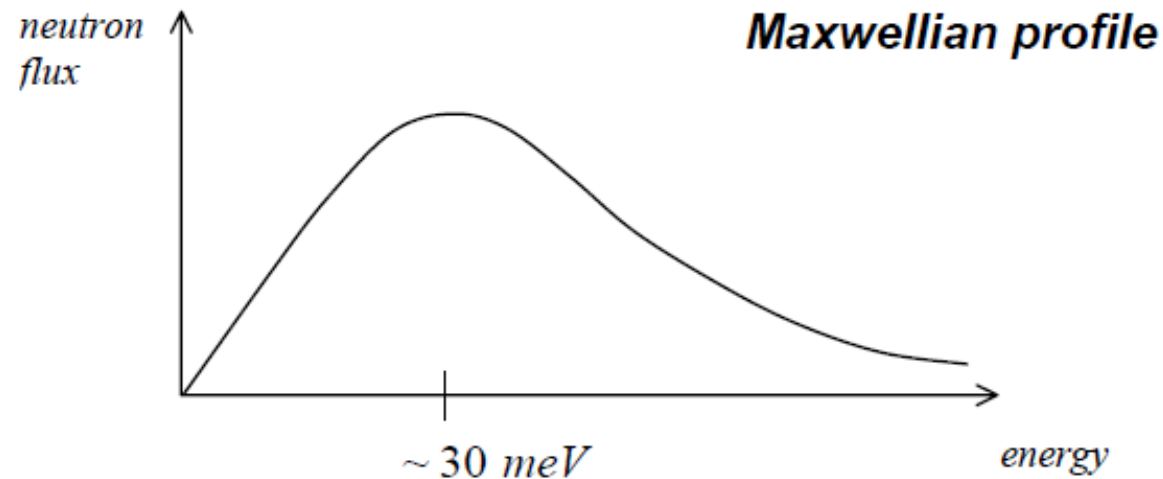
Outside view of the building (left) and layout of the experimental hall (right). The neutron beam tubes (blue) tap into the flux emitted from the reactor core (center) and guide the neutrons to various neutron scattering instruments. <http://www.frm2.tu-muenchen.de>

Research Reactors

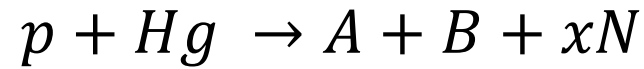
Research Reactors:



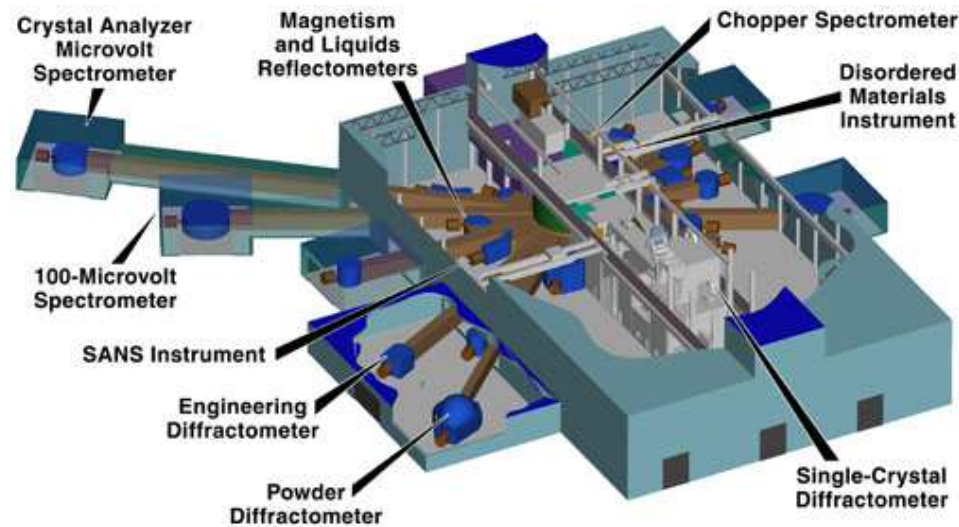
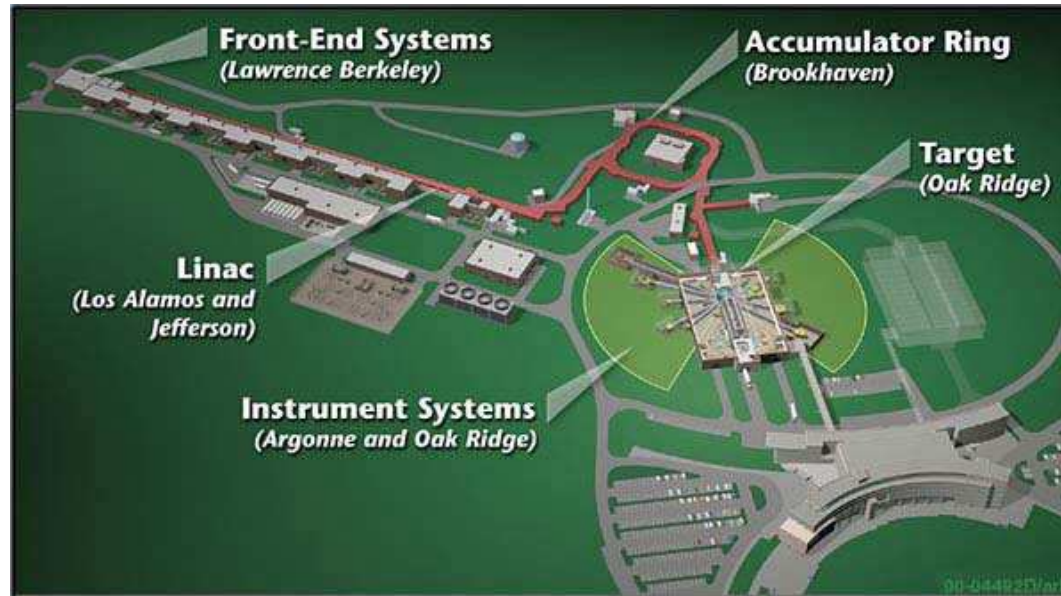
- Optimized for neutron flux => Low power (Research reactors are poor electrical generators)
- Fission is most favorable for thermal neutrons



Spallation Source



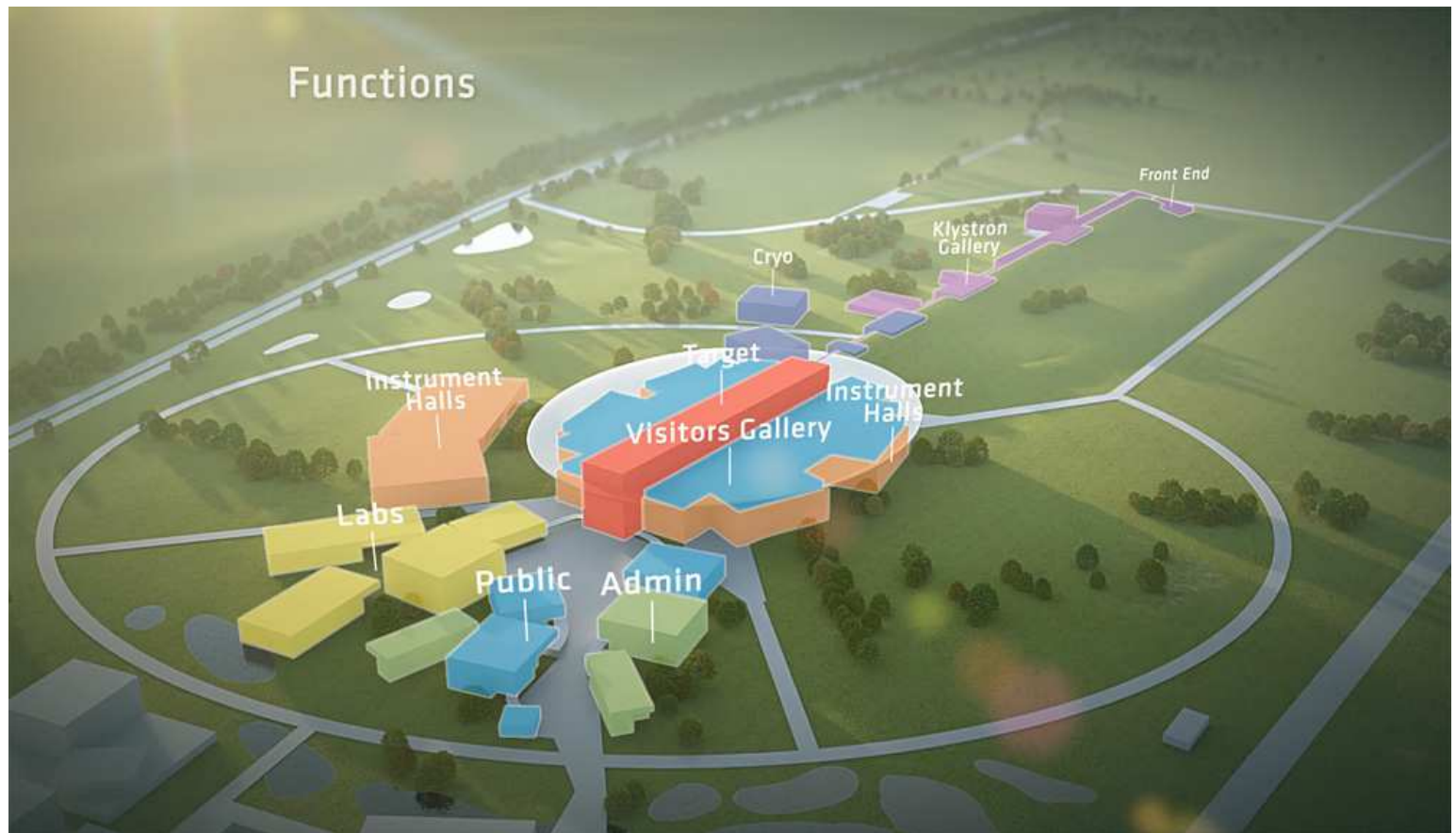
x = 20-30 !!



Spallation Source



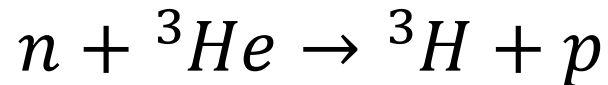
EUROPEAN
SPALLATION
SOURCE



Neutron Detectors

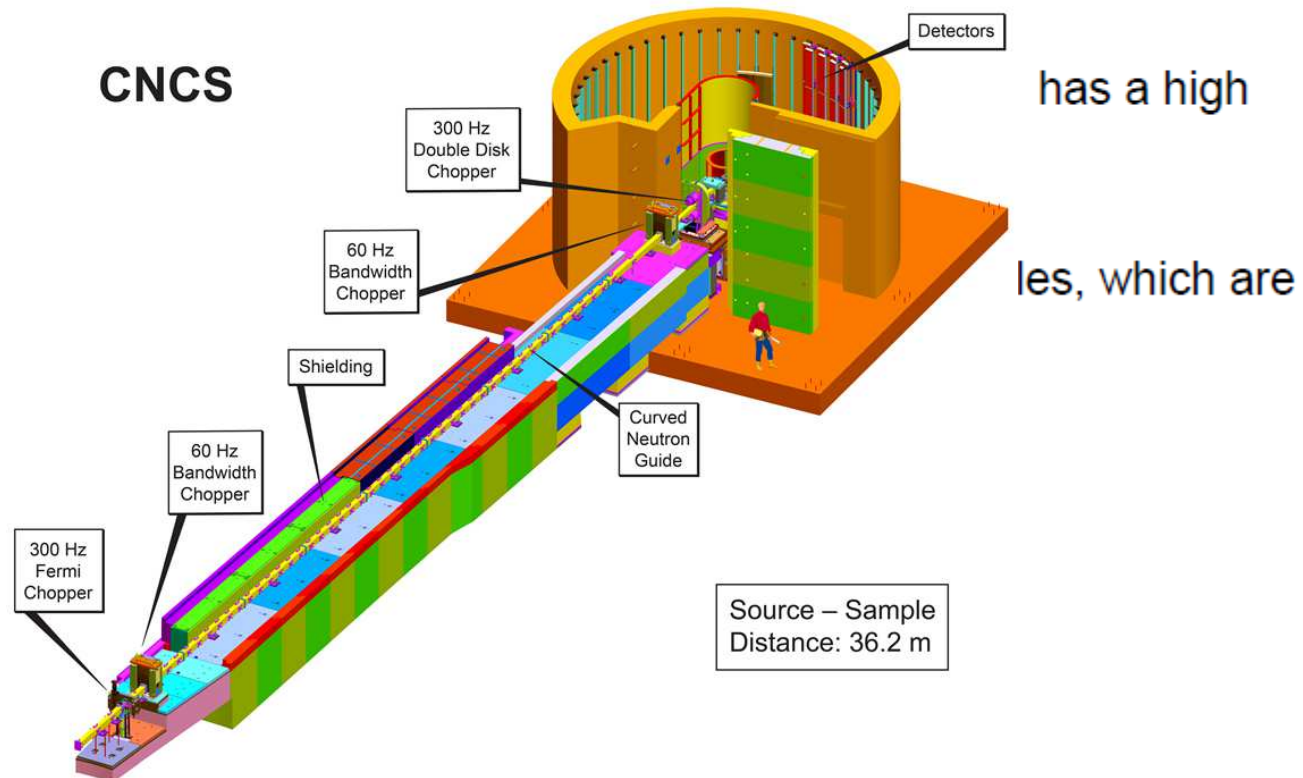
Neutrons have no charge and interact weakly with matter: hard to detect directly

Idea: convert them into charge particles



The protons are collected by a high electric field and converted into electric current.

Another type of neutron capture is $n + {}^{10}\text{B} \rightarrow {}^7\text{Li} + \alpha$. The energetic α particles are again collected.

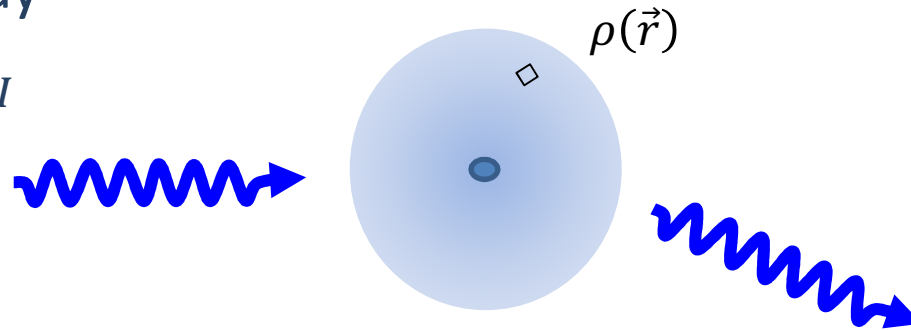


Neutron vs x-ray Scattering

Reminder

Incident x-ray

$$\hbar\omega_I, \vec{k}_I, \hat{\epsilon}_I$$



$$\frac{d\sigma}{d\Omega}(\vec{Q}) = r_0^2 (\hat{\epsilon}_F^* \cdot \hat{\epsilon}_I)^2 |F(\vec{Q})|^2 \quad \text{with } F(\vec{Q}) = \int \rho(\vec{r}) e^{i\vec{Q}\vec{r}} d\vec{r}$$

How did we get there ?

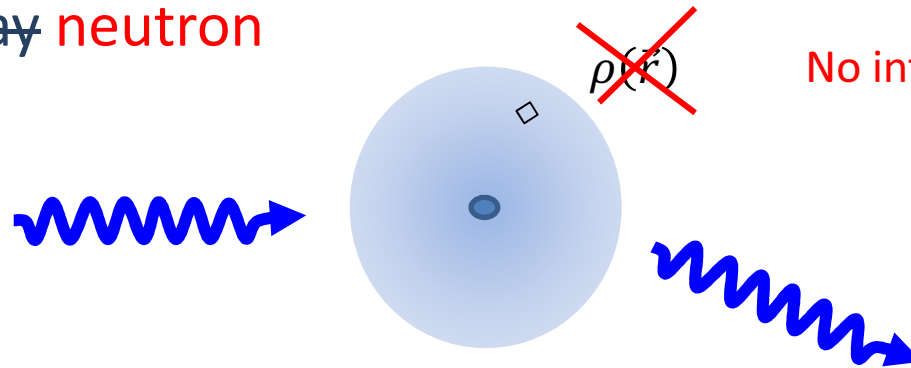
Thomson scattering from \diamond integrated over space

$$W_{I \rightarrow F} = \frac{2\pi}{\hbar} \left| \left\langle F \left| \frac{e^2 A^2}{2m} \right| I \right\rangle \right|^2 \rho(E_F)$$

Neutron vs x-ray Scattering

Incident ~~x-ray~~ neutron

$$E_I, \vec{k}_I$$



No interaction with electronic cloud

$$\frac{d\sigma}{d\Omega}(\vec{Q}) = r_0^2 \cancel{(\hat{\epsilon}_F^* \cdot \hat{\epsilon}_I)^2} |F(\vec{Q})|^2 \quad \text{with } F(\vec{Q}) = \int \rho(\vec{r}) e^{i\vec{Q}\vec{r}} d\vec{r}$$

How did we get there ?

~~Thomson scattering from~~ \diamond integrated over space

$$W_{I \rightarrow F} = \frac{2\pi}{\hbar} \left| \left\langle F \left| \frac{\cancel{e^2 A^2}}{2m} \right| I \right\rangle \right|^2 \rho(E_F)$$

Interaction of electrons with electro-magnetic field

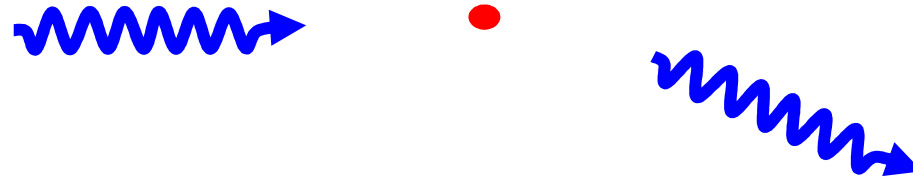
Neutron vs x-ray Scattering

$$W_{I \rightarrow F} = \frac{2\pi}{\hbar} \left| \left\langle F \left| \frac{2\pi\hbar^2}{m_n} b \delta(\vec{r} - \vec{R}) \right| I \right\rangle \right|^2 \rho(E_F)$$

Incident ~~x-ray~~ neutron

$$E_I, \vec{k}_I$$

Interaction of neutrons with nucleus: strong-force

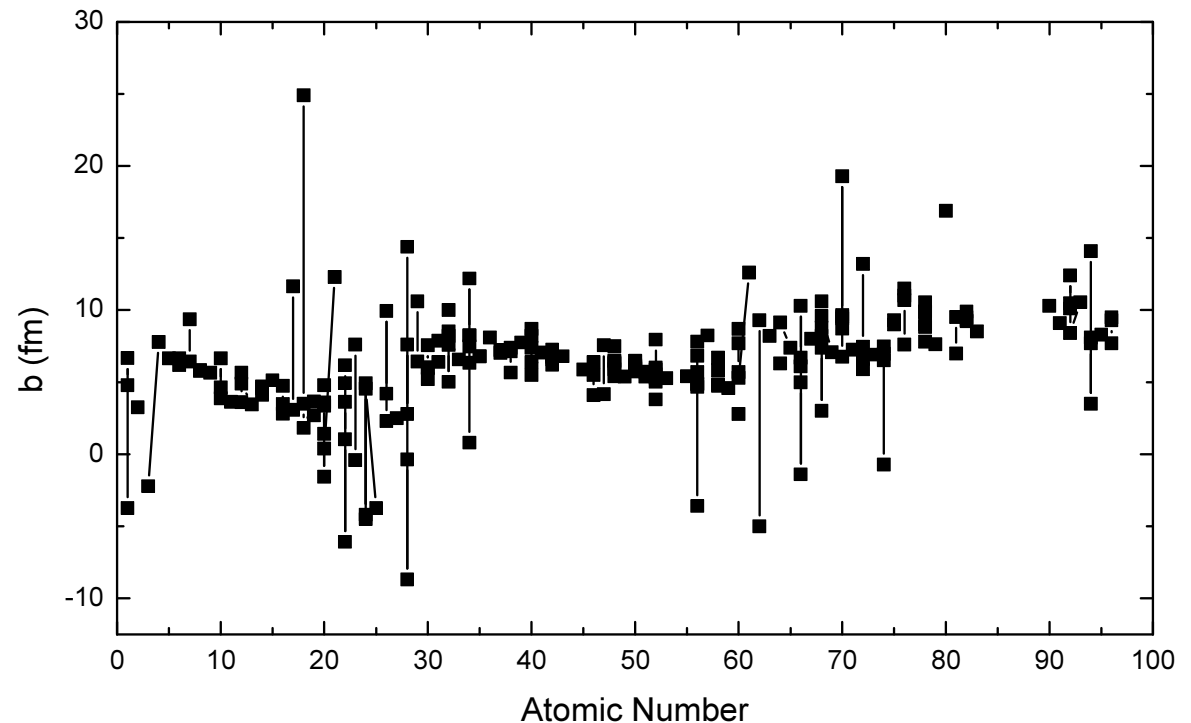


$$\frac{d\sigma}{d\Omega}(\vec{Q}) = ? \text{ See exercises}$$

Neutron Scattering length

b: scattering length

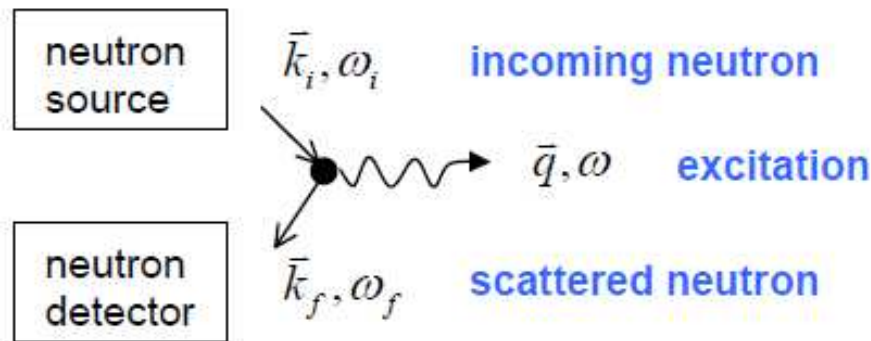
$$V(\vec{r}) = \frac{2\pi\hbar^2}{m_n} b \delta(\vec{r} - \vec{R})$$



<http://www.ncnr.nist.gov/resources/n-lengths/>

- 'Random' variation from nucleus to nucleus but also among isotopes !

Neutron Scattering Experiment



$$\vec{Q} = \vec{k}_f - \vec{k}_i$$

$$\omega = \omega_f - \omega_i = \frac{\hbar}{2m_n} (k_f^2 - k_i^2)$$

$$\omega = 0 \quad \text{elastic scattering}$$

$$\omega \neq 0 \quad \text{inelastic scattering}$$

Conceptually very similar to photons/x-rays !

=> Calculation of the scattering cross-section in the exercise class

BUT possibility of magnetic scattering
(resolution of magnetic structures)

Neutron vs x-ray Scattering

$$\left. \begin{aligned} |k_i\rangle &= \frac{1}{\sqrt{L^3}} e^{i\vec{k}_i \cdot \vec{r}} \\ |k_f\rangle &= \frac{1}{\sqrt{L^3}} e^{i\vec{k}_f \cdot \vec{r}} \end{aligned} \right\} \text{plane waves, normalized to sample size } L$$

$$\rho_f(E) = \underbrace{\left(\frac{L}{2\pi}\right)^3}_{\text{density of states in } k\text{-space}} \frac{d\vec{k}_f}{dE}$$

$$\text{for single nucleus: } \frac{d\sigma}{d\Omega} = |b|^2$$

In a crystal: If all nuclei are identical:

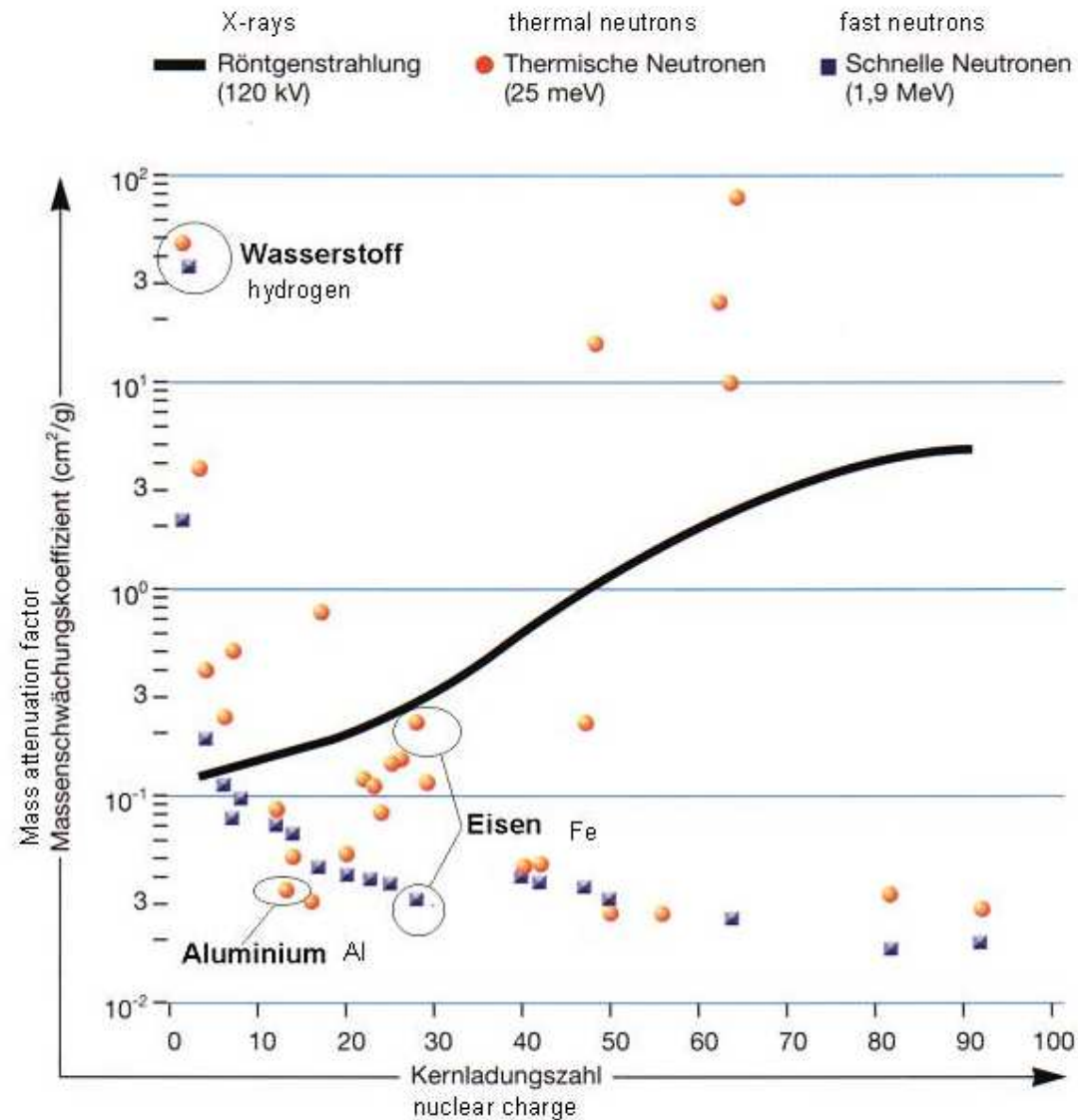
$$\frac{d\sigma}{d\Omega} = b^2 \frac{N(2\pi)^3}{v_0} \sum_{\vec{K}} \delta(\vec{Q} - \vec{K})$$

for unit cell with several atoms, basis vector \vec{d}

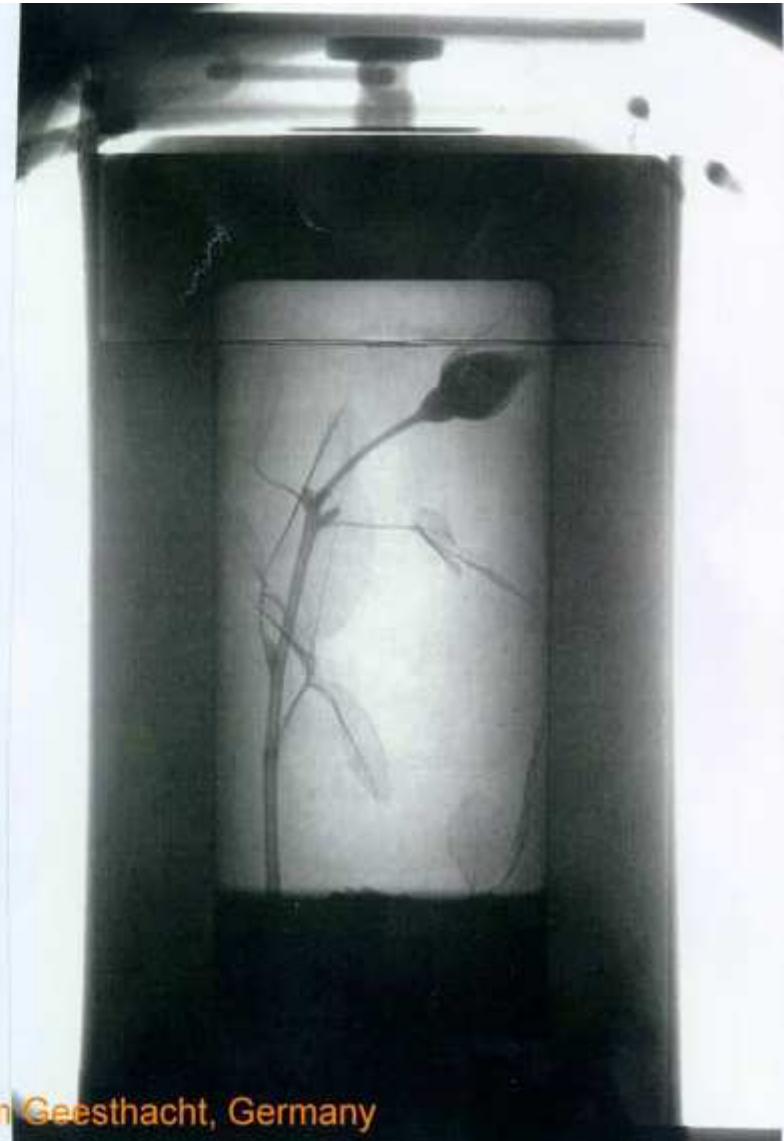
$$\frac{d\sigma}{d\Omega} = N \frac{(2\pi)^3}{v_0} \sum_{\vec{K}} \delta(\vec{Q} - \vec{K}) |F_N(\vec{K})|^2$$

$$F_N(\vec{K}) = \sum_{\vec{d}} e^{i\vec{Q} \cdot \vec{d}} b_{\vec{d}} \quad \text{“nuclear structure factor”}$$

Neutron Absorption

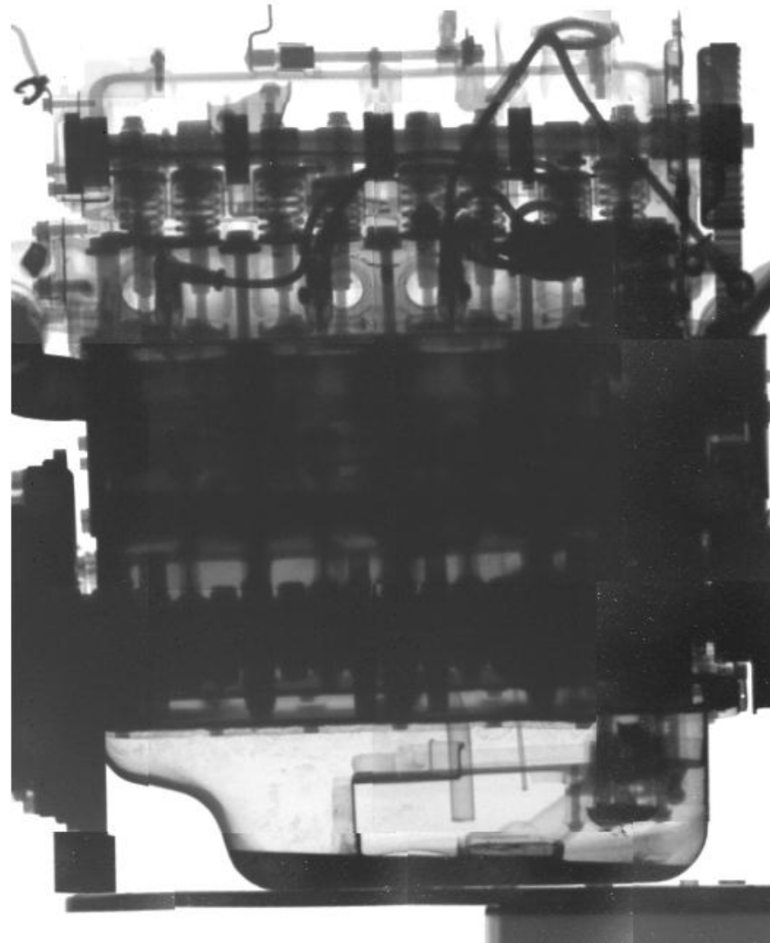


Neutron Absorption



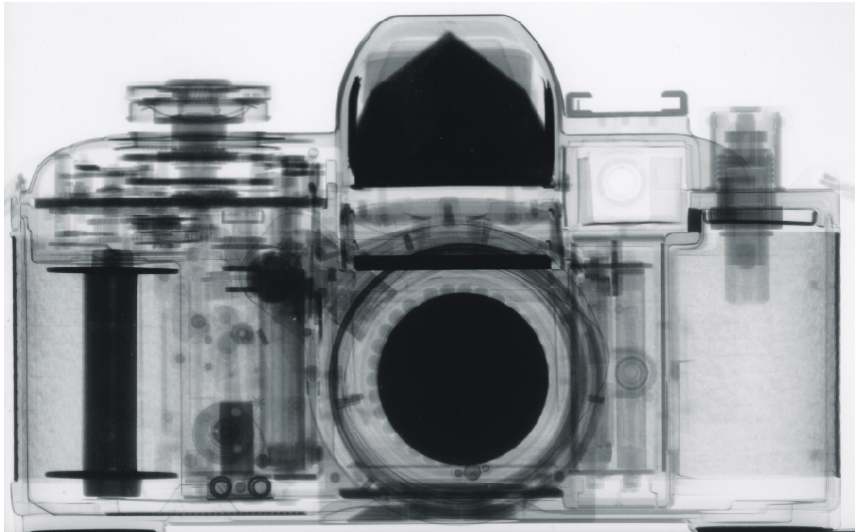
Courtesy L. Greim, GKSS, Forschungszentrum Geesthacht, Germany

Neutron Absorption

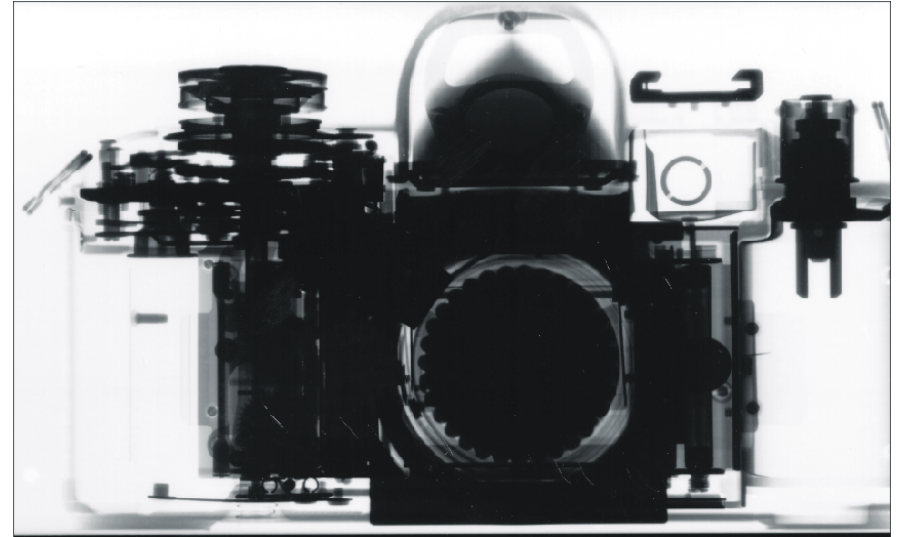


Neutron vs x-ray Radiography

Neutrons

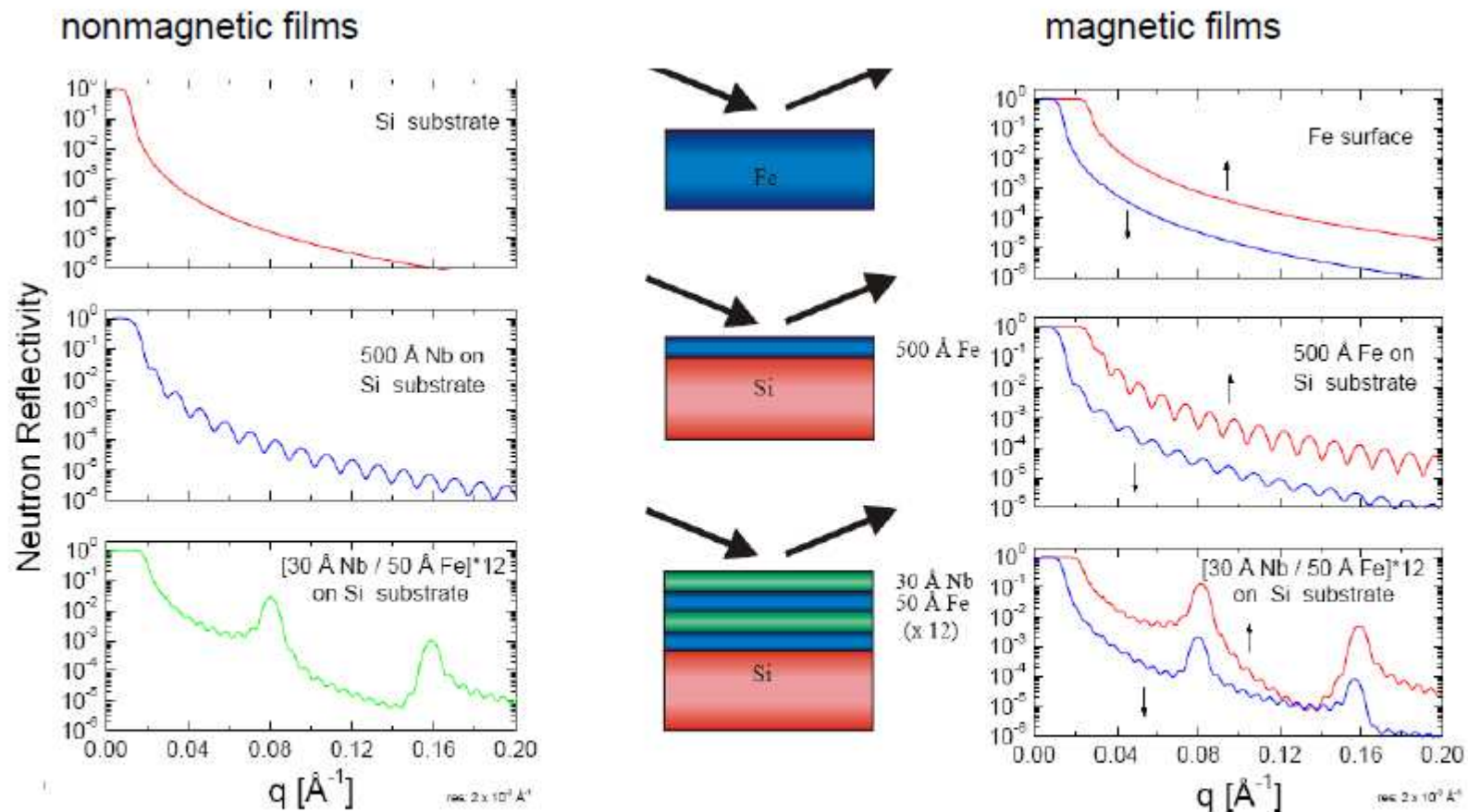


X-rays



<http://mnrc.ucdavis.edu/radiography.html>

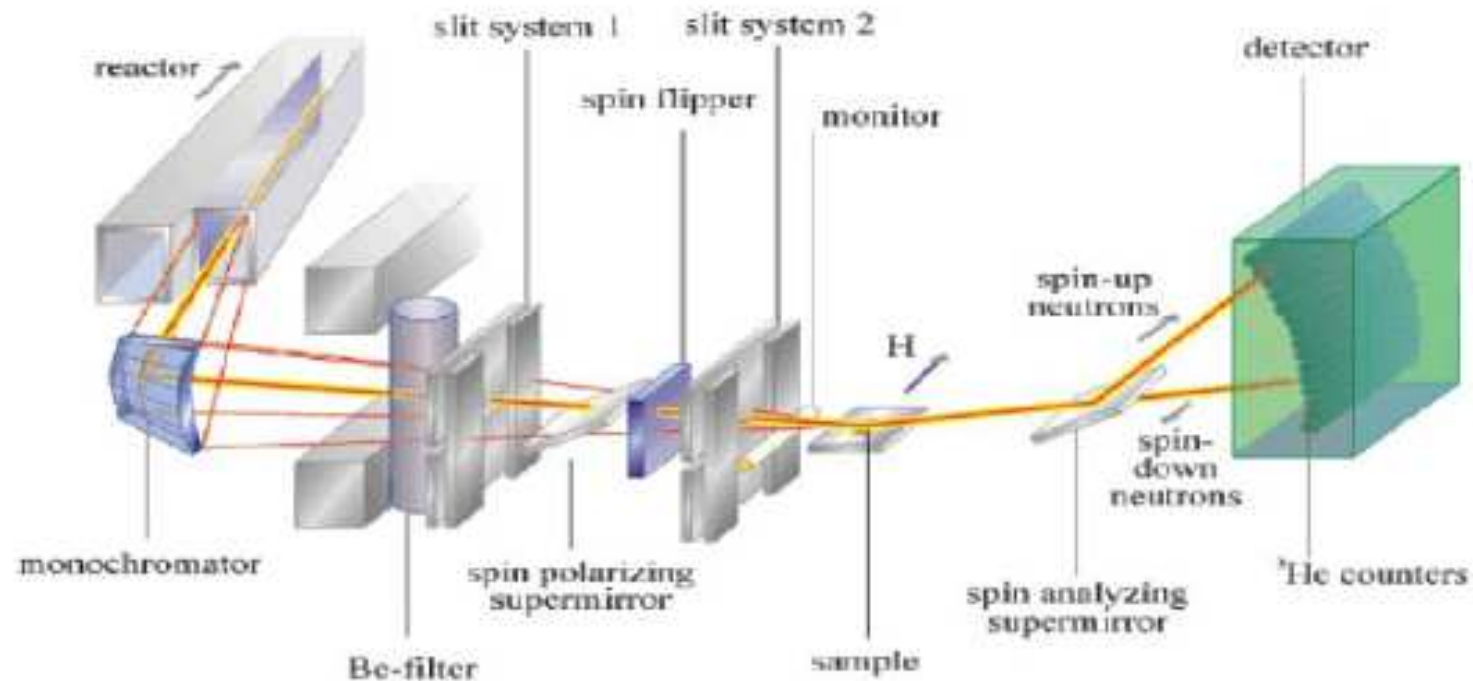
Ferromagnetic mirrors



Neutron reflectivity from nonmagnetic and magnetic films

<http://www.ornl.gov/council/02presentations/klose.pdf>

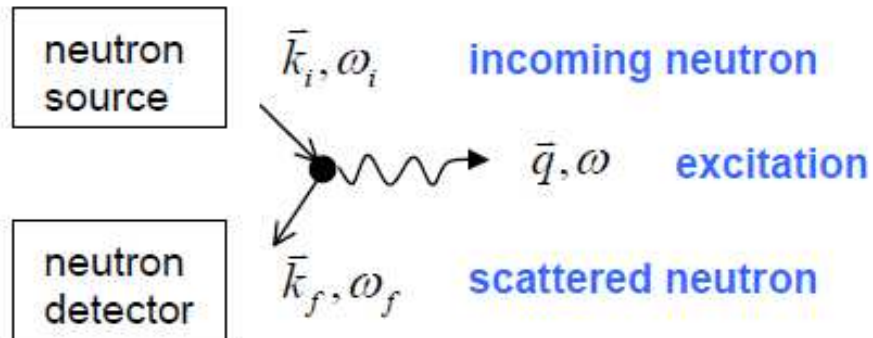
Ferromagnetic mirrors



Polarized-beam reflectometer

<http://www.ornl.gov/council/02presentations/klose.pdf>

Inelastic Neutron Scattering



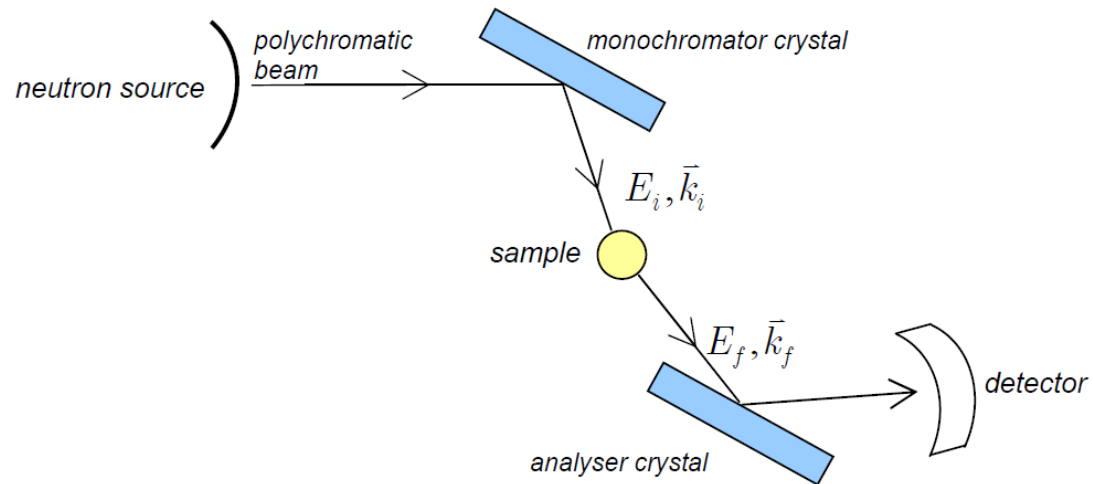
$$\bar{Q} = \bar{k}_f - \bar{k}_i$$

$$\omega = \omega_f - \omega_i = \frac{\hbar}{2m_n} (k_f^2 - k_i^2)$$

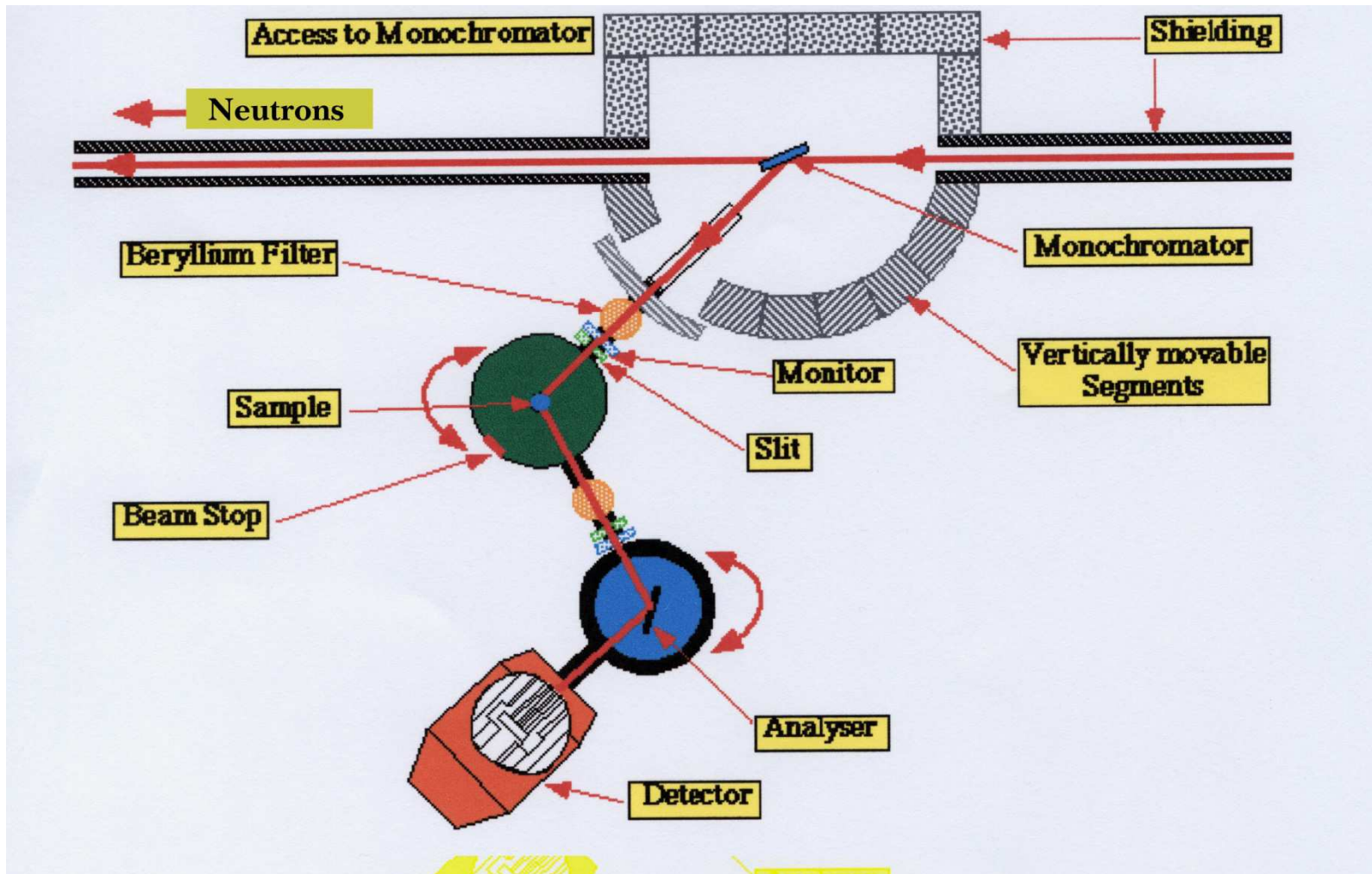
$\omega = 0$ elastic scattering

$\omega \neq 0$ inelastic scattering

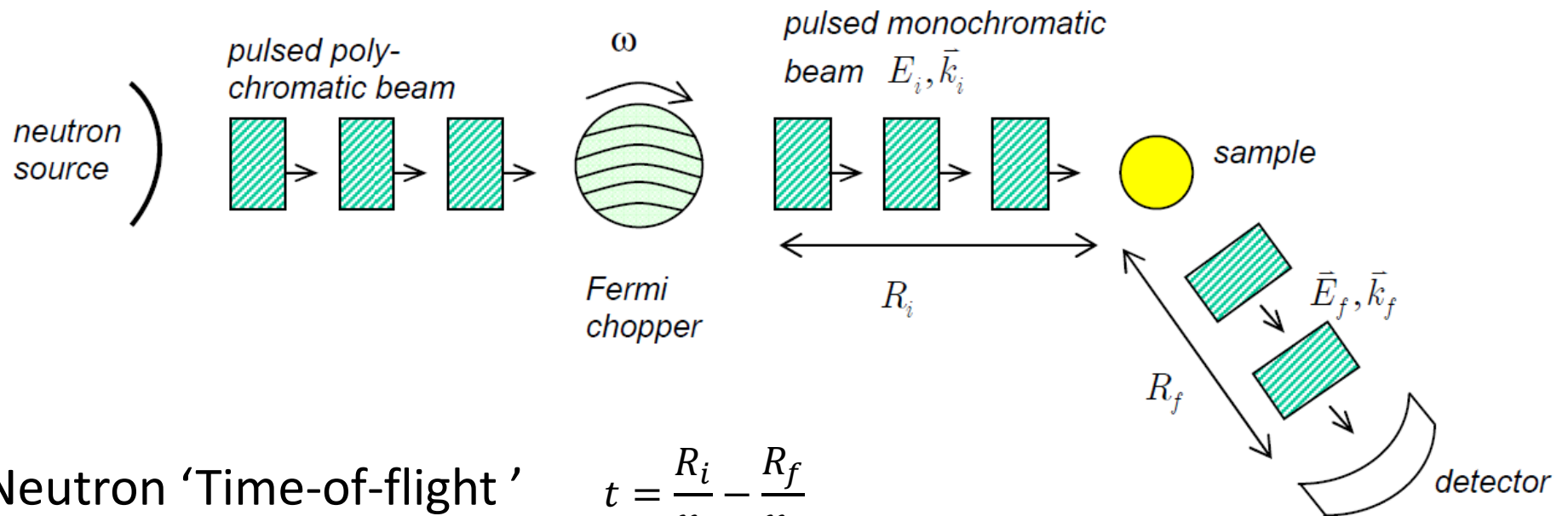
Triple Axis Spectrometer



Triple Axis Spectrometer



Time-of-Flight Spectrometer



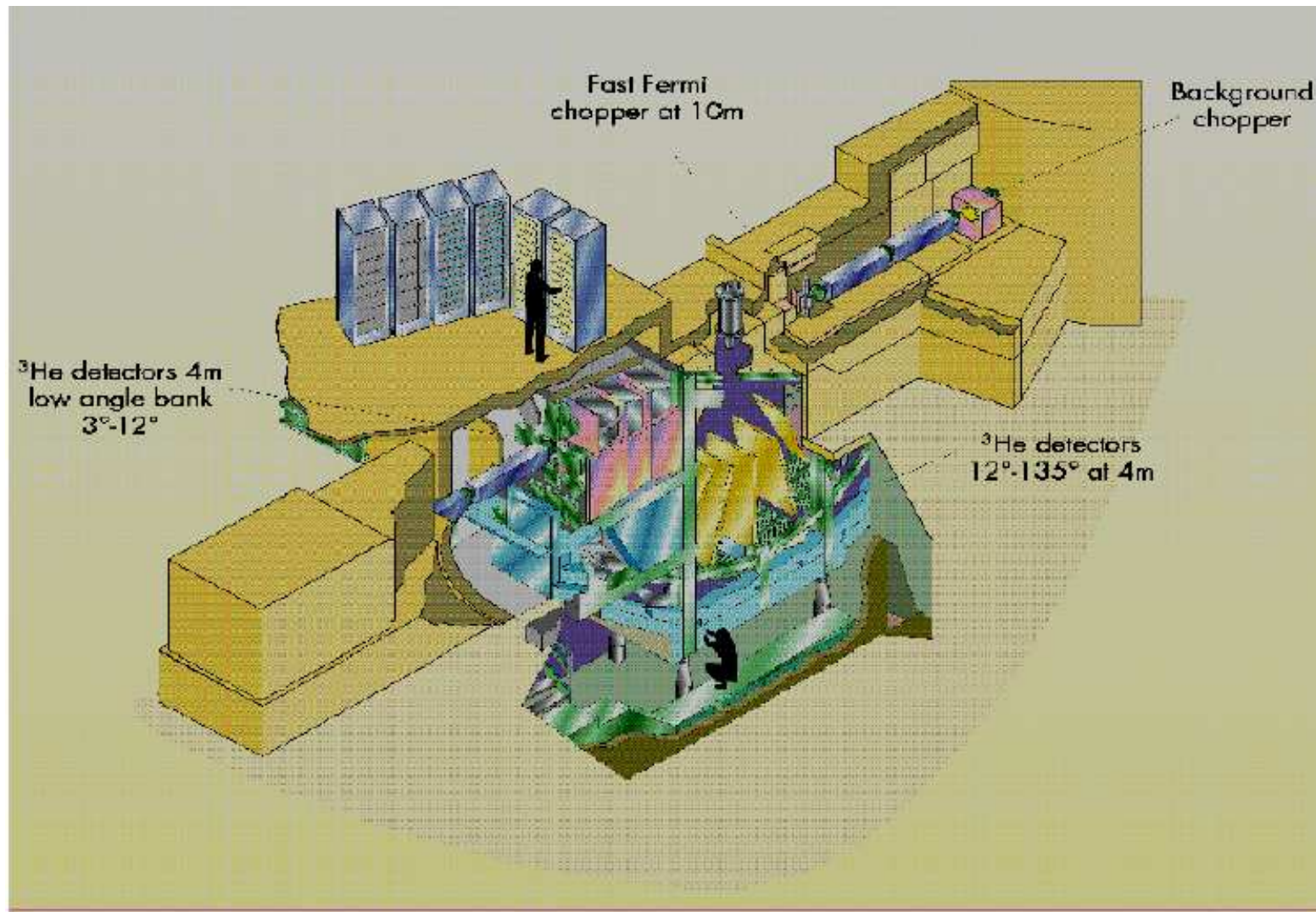
Neutron 'Time-of-flight'

$$t = \frac{R_i}{v_i} - \frac{R_f}{v_f}$$

$$v_f = \frac{R_f}{t - R_i/v_i}$$

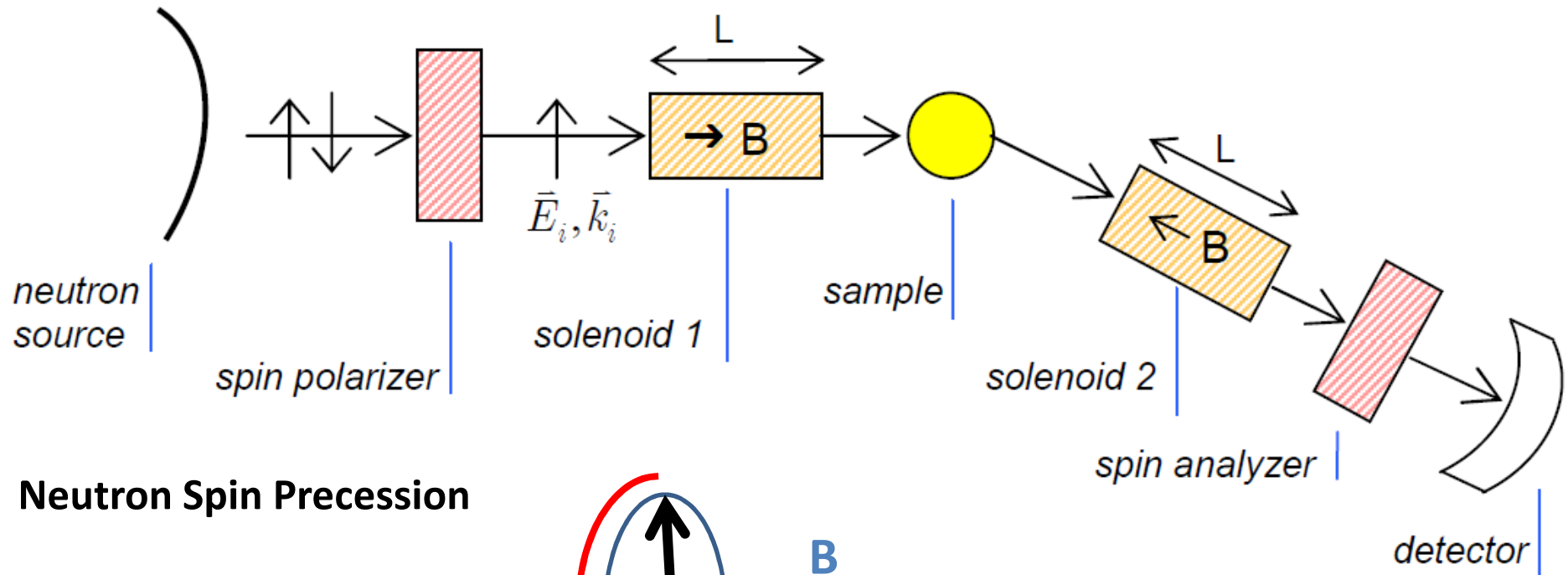
$$E_f = \frac{1}{2} m_N v_f^2 = \frac{1}{2} m_N \left(\frac{R_f}{t - R_i/v_i} \right)^2$$

Time-of-Flight Spectrometer

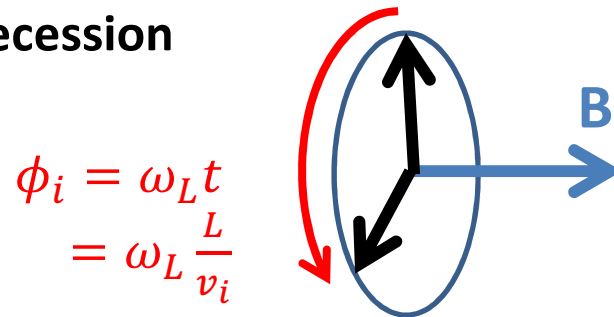


MARI Spectrometer @ISIS

Spin-Echo Spectrometer

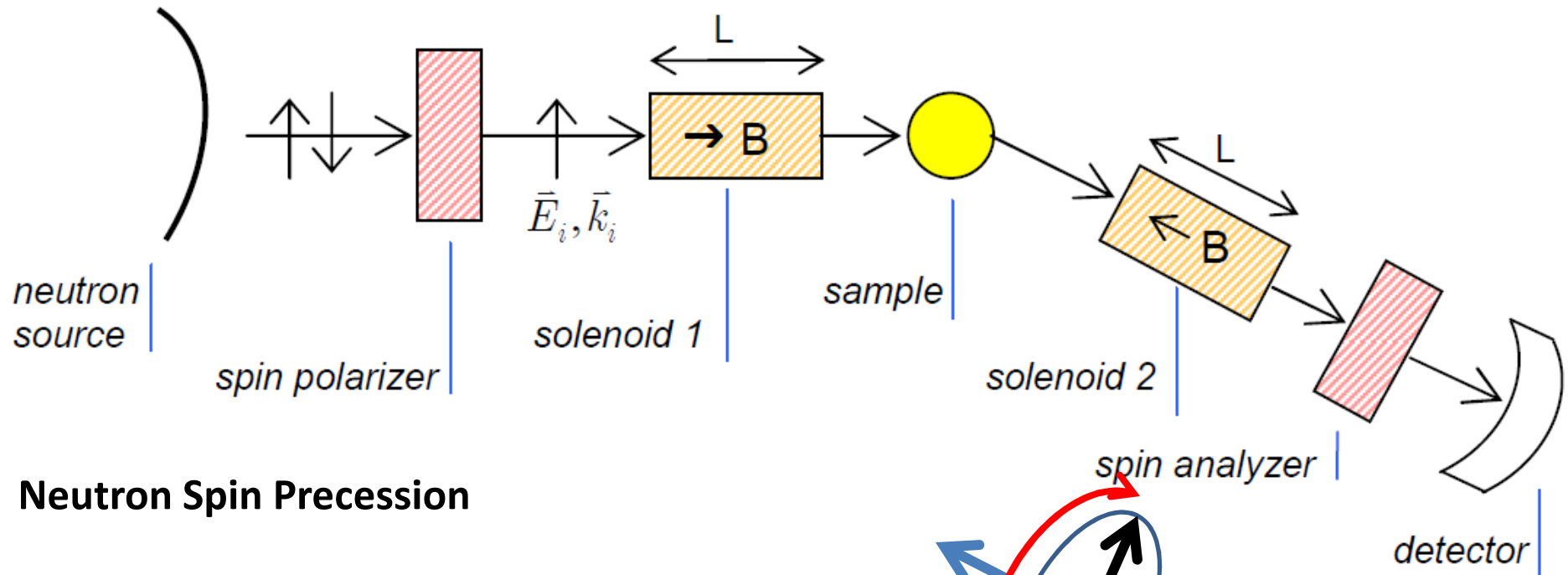


Neutron Spin Precession



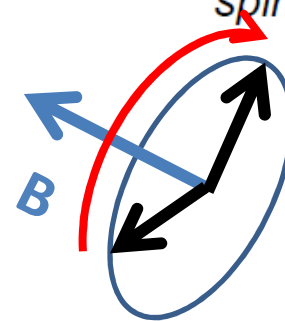
Larmor precession frequency $\omega_L = \frac{\gamma \mu_N B}{\hbar}$

Spin-Echo Spectrometer



Neutron Spin Precession

$$\begin{aligned}\phi_f &= -\omega_L t \\ &= -\omega_L \frac{L}{v_f}\end{aligned}$$



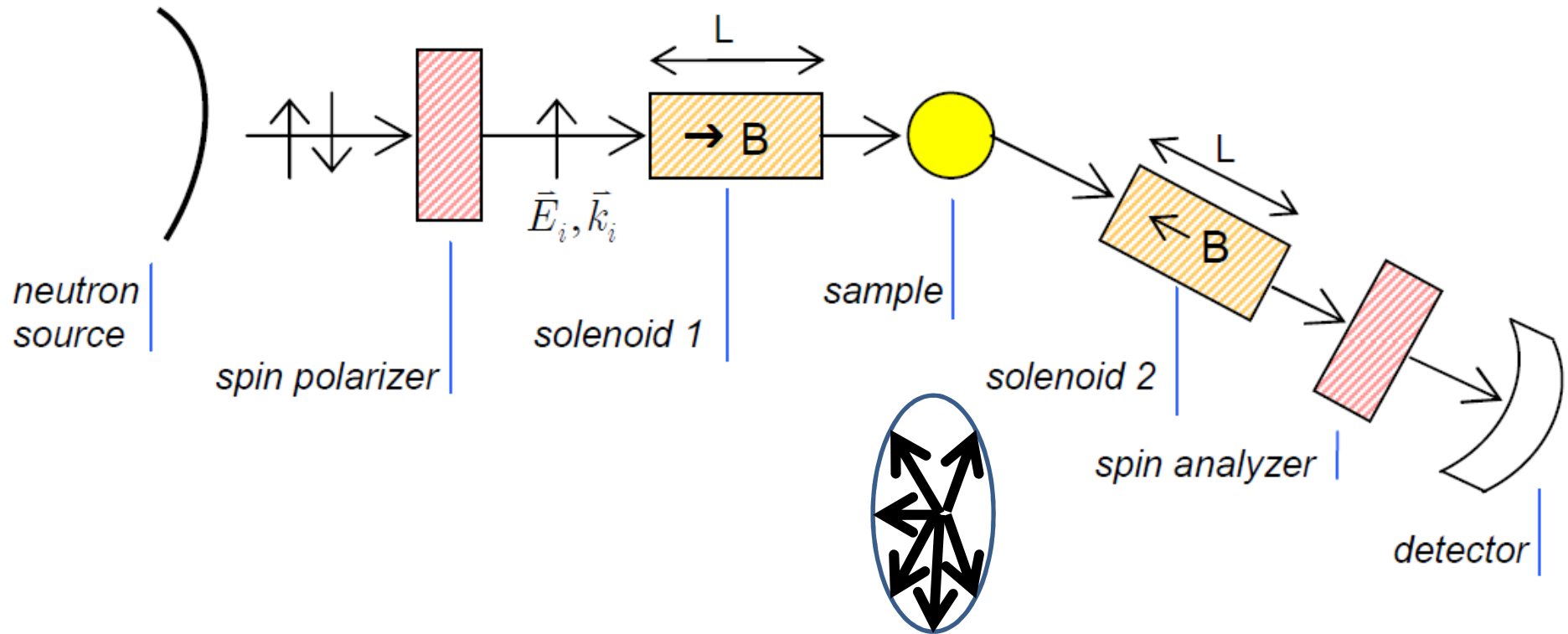
Back to initial polarization if $v_i = v_f$ (elastic scattering)

But shift $\Delta\phi$ if $v_f = v_i + \Delta v \neq v_i$ $\Delta\phi = \phi_i - \phi_f = \omega_L L \left(\frac{1}{v_i} - \frac{1}{v_i + \Delta v} \right) \sim \frac{\omega_L L}{v_i^2} \Delta v$

$$\hbar\omega = \frac{m_N}{2} (v_f^2 - v_i^2) = m_N \Delta v$$

$$\Delta\phi \sim \frac{\omega_L L}{v_i^3} \frac{\hbar\omega}{m_N} = \omega_L \tau_{SE}$$

Spin-Echo Spectrometer



NB: If the beam is not perfectly monochromatic (spread of v_i), it is depolarized at the sample position BUT recombination works the same !

Consequence: monochromaticity of the incident beam does not limit the experimental resolution that can be enhanced by order of magnitudes here !

Spin-Echo Spectrometer



NSE at NIST
http://www.ncnr.nist.gov/instruments/nse/NSE_70deg_20010226.png

Neutron scattering from quantum condensed matter

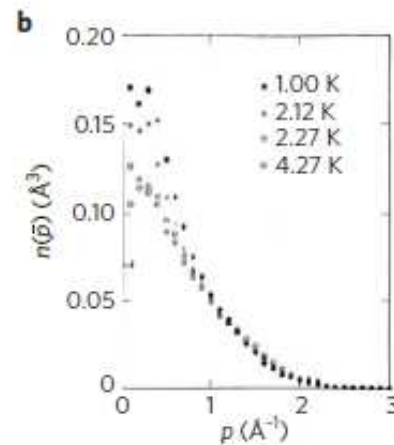
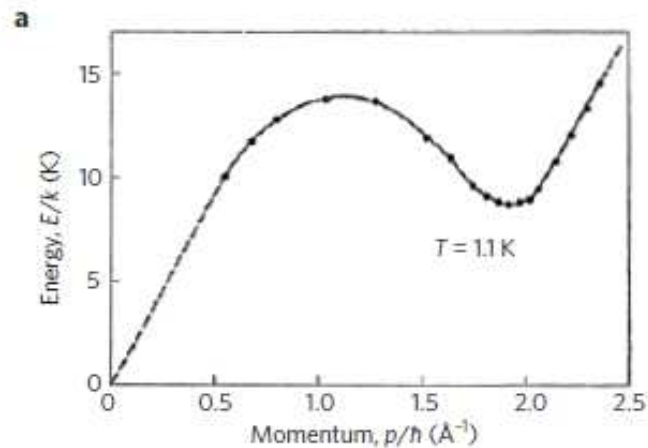
Steven T. Bramwell and Bernhard Keimer

Collective quantum phenomena such as magnetism, superfluidity and superconductivity have been pre-eminent themes of condensed-matter physics in the past century. Neutron scattering has provided unique insights into the microscopic origin of these phenomena.

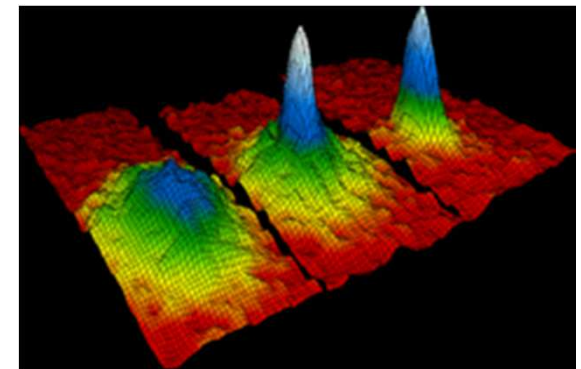
NATURE MATERIALS | VOL 13 | AUGUST 2014 | www.nature.com/naturematerials

763

Ex: Superfluid He

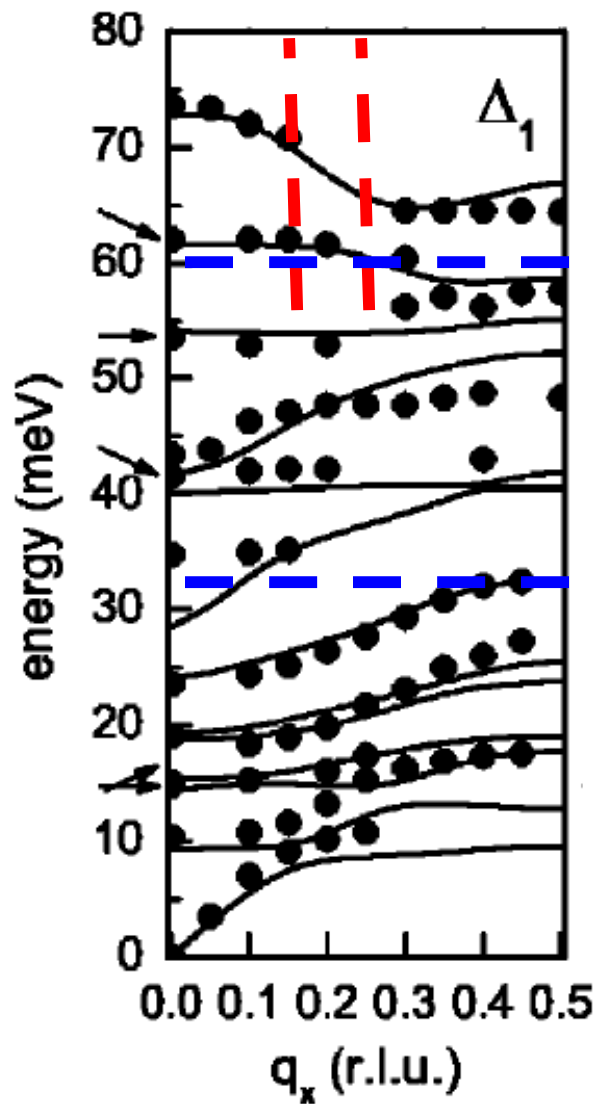


Analogy with BEC

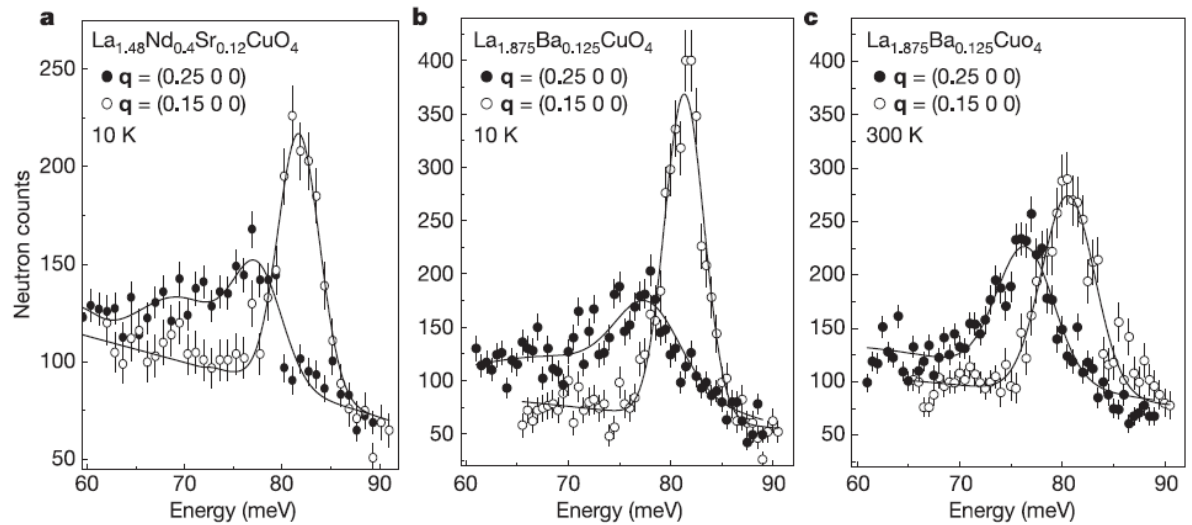


Triple Axis Spectrometer

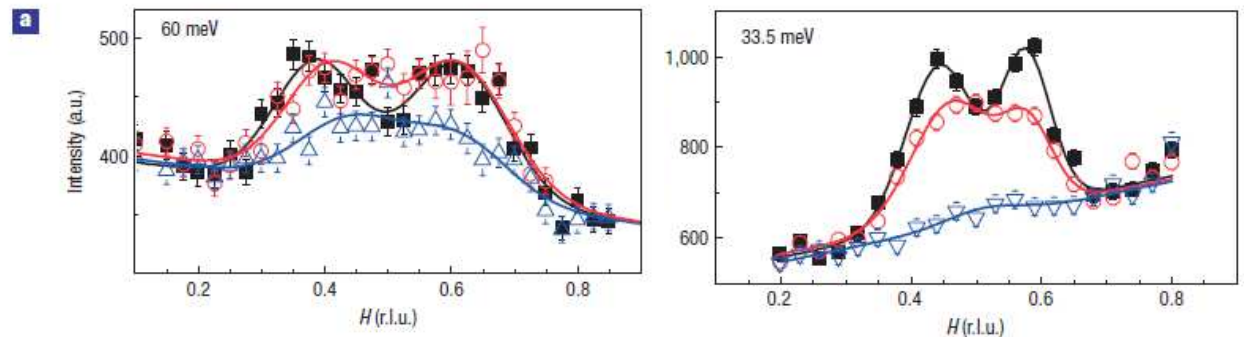
Dispersion Measurement



Constant-q scans

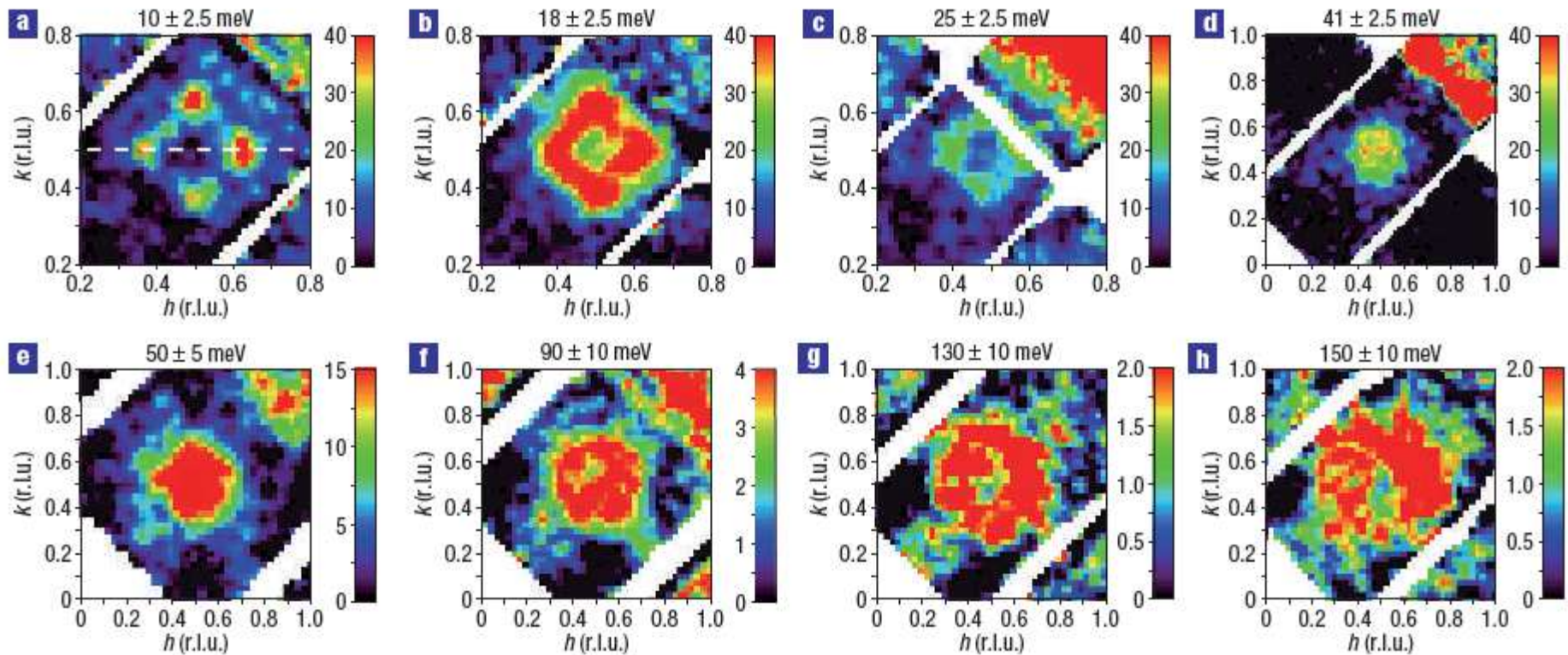


Constant-E scans (nb: here magnons)



Pintschovius et al. PRB 2004
Reznik et al. Nature 2006
Hinkov et al. Nature Physics 2007

Time-of-Flight Spectrometer



Reciprocal space maps at constant energy transfer

Vignolle et al. Nature Physics **3** 163 (2007)