Current-induced asymmetries in narrow quantum Hall systems

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Scanning force microscopy (SFM) has provided interesting information [1] about the position-dependence of Hall potential and current density in narrow Hall bars, with a width of about $10\,\mu$m. Under strong perpendicular magnetic fields B, which allow the observation of the integer quantized Hall effect (QHE), three types of the spatial variation of the Hall potential were observed. For B-values well outside the plateaus of the QHE, the Hall potential varied essentially linearly across the sample ("type I"), as one expects from the classical Drude theory. As the B-field entered a plateau region from the high-field side, a non-linear potential drop in a broad region in the middle of the sample was observed ("type II"). Most interestingly, for B-fields near the low-B edges of the plateaus the Hall potential was constant in a broad region in the center of the sample and dropped across two strips that moved with decreasing B towards the sample edges ("type III") [1]. Position and width of these strips coincided with those of the "incompressible strips" (ISs) expected to form in the sample due to the peculiar non-linear screening properties of the two-dimensional electron system in quantizing magnetic fields [2].

These experimental results have been qualitatively reproduced by a self-consistent calculation [3] of the screened confinement potential $V(\mathbf{r})$ and of the electron and current densities. Neglecting exchange and correlation effects and assuming that the confinement potential varies little over the spatial extent of a Landau state, electron density and conductivity tensor $\hat{\sigma}(\mathbf{r})$ were calculated for given $V(\mathbf{r})$ and position-dependent electro-chemical potential $\mu^\star(\mathbf{r})$ in the Thomas-Fermi approximation, assuming *local thermodynamic equilibrium*, and $V(\mathbf{r})$ was in turn calculated from the charge distribution by solving Poisson's equation. For given $\hat{\sigma}(\mathbf{r})$ the current density $\mathbf{j}(\mathbf{r})$ and the driving electric field $\mathbf{E}(\mathbf{r})$, produced by a stationary imposed current I, were calculated for a simple, in one direction translational invariant, Hall bar geometry under the assumption that the local form $\mathbf{j}(\mathbf{r}) = \hat{\sigma}(\mathbf{r})\mathbf{E}(\mathbf{r})$ of Ohm's law holds, with the gradient of $\mu^\star(\mathbf{r})$ as driving electric field, $\mathbf{E} = \nabla\mu^\star/e$. In this way $\mu^\star(\mathbf{r})$ depends on I, and the imposed current may have a non-linear feedback on electron and current density. Such a feedback was indeed obtained: The calculated ISs near the sample edges became with increasing strength of the imposed current increasingly asymmetric. The strip near the edge with intrinsic edge current in the direction of the imposed current became wider, while the width of the strip near the opposite edge became smaller [3]. Since this asymmetry vanishes in the linear-response limit of low imposed current [4], and since it was not clearly seen in the experiments [1], it was not investigated in detail.

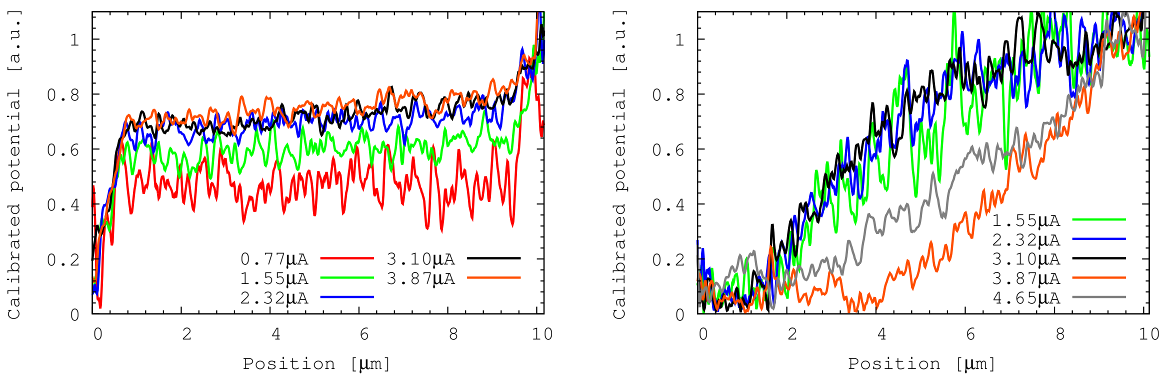

Figure 1: Normalized Hall potential in the "edge-dominated" region (left) and in the "bulk-dominated" region (right), for different imposed currents. Low temperature (1.4 K) measurements by K. Panos on narrow Hall bars ($10\,\mu$m width) made of GaAs/(AlGa)As heterostructures with electron concentration $n_{\mathrm{el}} = 2.9 \cdot 10^{15}\mathrm{m}^{-2}$. Magnetic field and Landau level filling factor are $B = 5.6\,$T, $\nu = 2.14$ (left) and $B = 6.1\,$T, $\nu = 1.97$ (right), respectively.

In recent years the SFM technique was employed in the group of J. Weis to investigate several interesting problems. One of these problems is the distribution of current and Hall potential in narrow quantum Hall systems under strong imposed currents, which lead to the breakdown of the QHE. Will this breakdown occur suddenly, as a kind of phase transition, or will there be a continuous change from a non-dissipative to a dissipative state? K. Panos has carried out many low-temperature experiments on narrow Hall bars made of GaAs/(AlGa)As heterostructures (similar experiments on graphene will be discussed separately) and found essentially two different situations, which are sketched in Fig. 1. One situation was met for B-fields well within the plateaus of the QHE and near their low-B edges. In these "edge-dominated" regions the Hall potential profile changes continuously with the strength of the imposed current I. For small I the (normalized) Hall potential $U_H(x)$ drops across strips near the sample edge and is flat in between. (The noisy fluctuations are due to the complicated measuring technique and become smaller with increasing current.) This corresponds to the "type III" behavior observed in Ref. [1]. With increasing I, steps of $U_H(x)$ near the edges become very asymmetric, and finally a linear variation of $U_H(x)$ in the bulk region between the steps occurs (see left part of Fig. 1). The right part of Fig. 1 shows an example of the other situation. In this "bulk-dominated" region the B-field is closely above a high-B edge of a QH plateau and, for small I, $U_H(x)$ varies in a more or less wide stripe in the center of the sample ("type III" behavior of Ref. [1]), and the shape of this variation is nearly unchanged. At a certain critical value of the imposed current, however, there occurs a sudden and abrupt change of the $U_H(x)$ profile, and the shape of the $U_H(x)$ curve depends on the position along the sample where the scan across the sample is performed.

In view of these new experiments the asymmetry predictions of Ref. [3] become interesting, and we have performed similar self-consistent calculations in order to investigate to what extent the model used in Ref. [3] can explain the recent experiments. We have changed the model of Ref. [3] essentially in two points. First, in Ref. [3] a cut-off $\sigma_l(x) > \epsilon \cdot \sigma_H(x)$ with $\epsilon \sim 10^{-4}$ was used for x-values in the ISs, in order to avoid divergencies. We found however, that for relevant values of the temperature and of the collision broadening of Landau levels this cut-off becomes effective and determines the position-dependence of the current density. So we avoided this cut-off. Second, we followed the discussion of Ref. [4] and replaced the conductivity tensor $\hat{\sigma}(x)$ calculated within the local thermal equilibrium approach by a smoothed form $\hat{\bar{\sigma}}(x) = \int_{-\lambda}^{\lambda} d\xi\, \hat{\sigma}(x+\xi)/2\lambda$, with x the position across the Hall bar and λ a length of the order 10 to 20 nm. This replacement repairs to some extent physical and mathematical problems resulting from the oversimplifying assumptions of vanishing spatial extent of the Landau states and of a strictly local form of Ohm's law [4]. (This replacement, e.g., avoids that arbitrarily narrow ISs in the limit of very low temperature can lead to vanishing longitudinal resistance [4].) For the calculation of the conductivity and the electron density we use Gaussian spectral functions $A_n(E) = \exp(-[E_n - E]^2/\Gamma^2)/(\sqrt{\pi}\,\Gamma)$ with Landau energies $E_n = \hbar\omega_c(n + 1/2)$, as in Ref. [3], with $\Gamma = \gamma\, \hbar\omega_c[10\mathrm{T}/B]^{1/2}$ and γ a measure of collision broadening. Due to the unavoidable intrinsic fluctuations of the experimental data (see Fig. 1 and Ref. [1]) our aim cannot be to reproduce the experimental results quantitatively, but only to understand them qualitatively. Therefore we try to keep the computational time short and choose small sample width ($2d = 3\,\mu$m) and a small number of mesh points in the integrations ($N \sim 500$) (for $N \sim 1000$, and for $2d = 5\,\mu$m, the results are qualitatively the same).

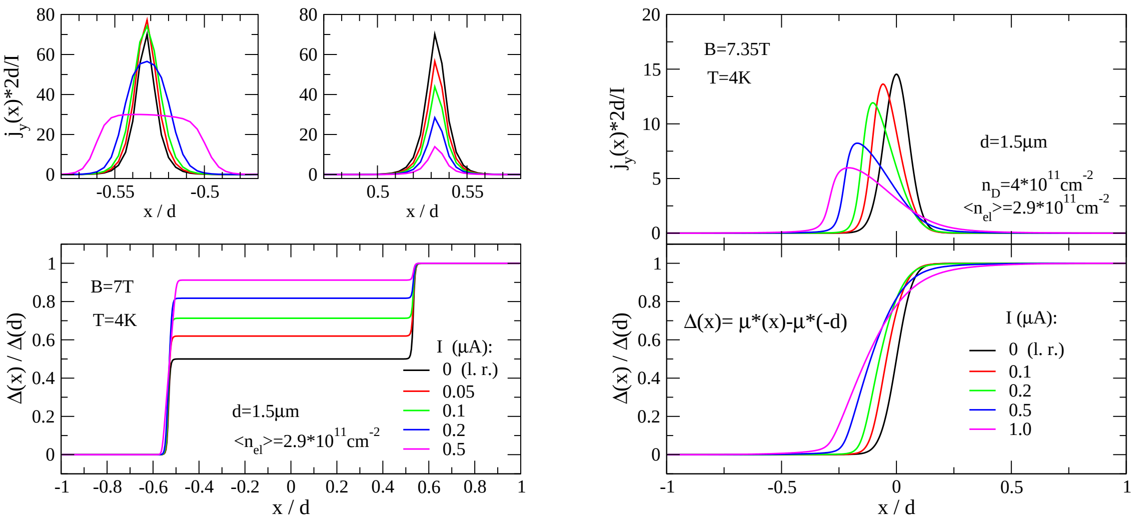

Figure 2: Normalized Hall potential $U_H(x) = [\mu^\star(x) - \mu^\star(-d)]/[\mu^\star(d) - \mu^\star(-d)]$ across (lower parts of the figures) and relevant parts of the imposed current density $j_y(x)$ along (upper parts) a $2d = 3.0\,\mu$m wide Hall bar with translational invariance in y-direction, for $B = 7.0$ T (left) and $B = 7.35$ T (right side), respectively; calculation for $T = 4$ K, $\gamma = 0.01$, $\lambda = \ell_B = [10\mathrm{T}/B]^{1/2} \cdot 8.11$ nm, and several values of the imposed current I. The Landau level filling factors $\nu(x = 0)$ in the center are 2.10 (left) and 2.00 (right), respectively.

Figure 2 shows, for relatively small imposed currents I, typical results for the "edge-dominated" and the "bulk-dominated" case in the left and the right parts, respectively. The model parameters are chosen so that for $I = 0$, i.e. in thermal equilibrium, electron density and confinement potential are symmetric. This leads, for finite I, to the symmetry relations $j_y(x; -I) = -j_y(-x; I)$ for the current density along the Hall bar and to $U_H(x; -I) = 1 - U_H(-x; I)$ for the normalized Hall potential. In the linear response regime, $I \to 0$, this implies obvious symmetries of the $j_y(x; I)/I$ and $U_H(x; 0)$ curves. The left part of Fig. 2 describes a situation with two well developed incompressible strips, which carry practically all the imposed current dissipationless, and lead to the steps in the Hall potential, whereas $j_y(x)$ practically vanishes and $U_H(x)$ is flat in the dissipative compressible regions. With increasing I, the $j_y(x)$ peaks in the ISs and the heights of the corresponding steps of $U_H(x)$, which are symmetric in the linear response limit, become increasingly asymmetric. The right part of Fig. 2 describes a situation with a single IS in the center. For small increasing I the increasing asymmetry of the Hall potential is similar to that observed in the right part of Fig. 1. If we slightly increase B, we get $\nu(x) < 2$ everywhere and only the lowest Landau level is partly occupied (we neglect spin splitting). Then the dissipation in the center increases rapidly to the bulk value, and the current spreads out rapidly. The Hall potential shows the "type II" behavior of Ref. [1] and is "bulk-dominated". With further increasing B the current spreads out over the whole sample and $U_H(x)$ approaches the linear Drude behavior ("type I"). This is shown in the left part of Fig. 3.

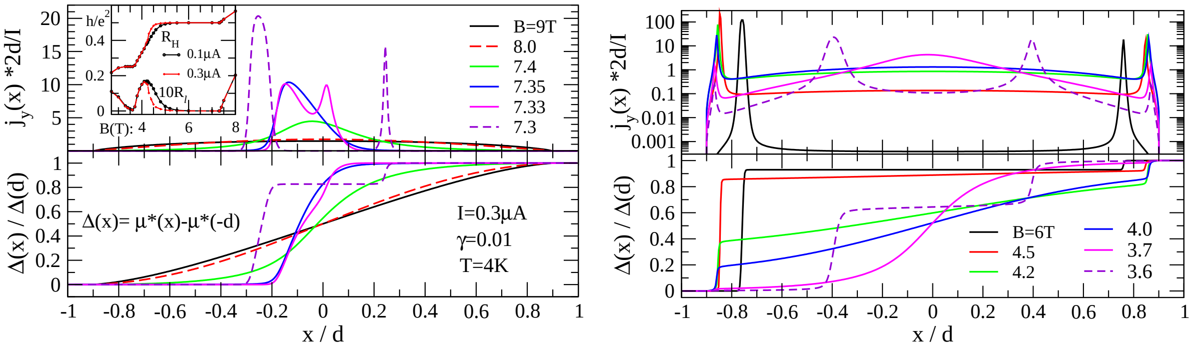

Figure 3: Normalized Hall potential (lower parts) and current density $j_y(x)$ (upper parts) calculated for several values of the magnetic field, assuming spin-degeneracy. The inset in the upper left shows the calculated Hall and longitudinal resistances (in units of h/e^2) for $I = 0.1\,\mu$A and $I = 0.3\,\mu$A. Other parameters as in Fig. 2. Note the logarithmic scale in the upper right figure.

For B-values slightly below $B = 7.35\,$T the central IS splits into two ISs, which with decreasing B move towards the sample edges and become narrower. If they become too narrow, the current density spreads out into the dissipative compressible bulk region, and the Hall potential gets a finite slope in the region between the steps. With further decreasing B the steps become smaller and $U_H(x)$ approaches the linear "type I" behavior, until the next QH plateau is reached (here with $\nu(0) = 4$ at $B \approx 3.7\,$T). All these theoretical results are in nice qualitative agreement with the recent experiments by K. Panos and show that, in these narrow Hall bars, the "breakdown" of the QHE as function of B is a continuous process.

There are, however, also theoretical results, which are not confirmed by the experiments. We find that with increasing I the IS carrying the larger amount of current, and thus leading to the larger step in the Hall potential, becomes wider. There are good electrostatic reasons, which support this finding [2], but it is not clearly seen in the experiments due to the limited spatial resolution. A serious shortcoming of our theory is the lack of breakdown mechanisms. At not too low temperatures a part of the current will flow through dissipative regions. Then Joule heating will increase the electron temperature and possibly lead to a breakdown of the QHE. Another possible breakdown mechanism is quasi-elastic inter-Landau-level-scattering (QUILLS), which should become possible in ISs, which carry larger currents. This follows again from the electrostatic arguments [2], which say that the potential change ΔV across an IS, which increases linearly with the current through the IS, leads to a strip width $w \propto \sqrt{\Delta V}$. Then the distance of isoenergetic states of adjacent Landau levels in the strip decreases as $\hbar\omega_c \cdot w/\Delta V \propto 1/\sqrt{\Delta V}$. QUILLS may be the reason for a current-induced breakdown of the QHE, which is not seen in our calculation. Therefore it seems necessary to consider Joule heating and QUILLS in the theoretical approach.

References:

[1] *Ahlswede, E., P. Weitz, J. Weis, K. von Klitzing, and K. Eberl.* Physica B **298**, 562 (2001).

[2] *Chklovskii D.B., B.I. Shklovskii, and L.I. Glazman.* Phys. Rev. B **46**, 4026 (1992).

[3] *Güven K. and R.R. Gerhardts.* Phys. Rev. B **67**, 115327 (2003).

[4] *Siddiki A. and R. R. Gerhardts.* Phys. Rev. B **70**, 195335 (2004).