Singular order parameter interaction at nematic quantum critical point

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Numerous correlated electron compounds undergo a quantum phase transition between ground states with different symmetries, which can be tuned by a non-thermal control parameter such as doping, pressure, or a magnetic field. In the vicinity of a continuous transition electronic excitations are strongly scattered by order parameter fluctuations. Quantum critical fluctuations near a quantum critical point (QCP) are therefore frequently invoked as a mechanism for non-Fermi liquid behavior in strongly correlated electron compounds.

Quantum criticality in metallic electron systems is traditionally described by an effective order parameter theory, which was pioneered by Hertz and Millis. In that approach, an order parameter field ϕ is introduced via a decoupling of the electron-electron interaction, and the electronic variables are subsequently integrated out. The resulting effective action $S[\phi]$ for the order parameter is truncated at quartic order and analyzed by standard scaling techniques. However, several studies revealed that the Hertz-Millis approach may fail, especially in low-dimensional systems. Since electronic excitations in a metal are gapless, integrating out the electrons may lead to singular interactions between the order parameter fluctuations which cannot be approximated by a local quartic term. A break-down of Hertz-Millis theory is known to occur for an antiferromagnetic quantum phase transition in two dimensional metals, and for a ferromagnetic transition even in three dimensions.

Here we investigate whether the Hertz-Millis approach is valid for two-dimensional systems exhibiting a quantum phase transition driven by *forward scattering* in the charge channel. The most prominent such transition is the electronic *nematic*, in which an orientation symmetry is spontaneously broken, while translation and spin-rotation invariance remain unaffected. The problem of quantum critical points with singular forward scattering is closely related to the problem of non-relativistic fermions coupled to a $U(1)$ gauge field. Calculations in the gauge field context suggested that the simple form of the (bosonic) fluctuation propagator obtained in lowest order (RPA) remains unaffected by higher order terms. In a recent paper Metlitski and Sachdev [1] formulated a scaling theory of the nematic QCP and related problems, treating the electrons and order parameter fluctuations on equal footing. No qualitative correction was found for the fluctuation propagator, up to three-loop order. This is in stark contrast to the case of an antiferromagnetic QCP in two dimensions, where the fluctuation propagator is substantially renormalized compared to the RPA form. A clarification of the properties of the nematic QCP beyond three-loop order is still lacking.

The robustness of the fluctuation propagator at the nematic QCP seems to indicate that interactions of the order parameter fluctuations are irrelevant such that the QCP is Gaussian, in agreement with the expectations from Hertz-Millis theory. It is therefore worthwhile to analyze the interaction terms in the effective action $S[\phi]$ obtained after integrating out the electrons. The N-point interactions are given by fermionic loops with N vertices. To obtain the scaling behavior of such loops is non-trivial, because the most naive power-counting is easily invalidated by cancellations [2]. We have computed the exact scaling behavior of the N-point interactions for the nematic QCP and related systems, and found that the interactions are *marginal* and *non-local* for all $N \geq 3$. Hence, replacing them by a local ϕ^4 interaction is not justified [3]. In the following, we sketch the main points of the theory.

We consider an interacting electron system which undergoes a continuous quantum phase transition with a scalar order parameter of the form

$$O = \sum_\sigma \int \frac{d^2k}{(2\pi)^2}\, d(\mathbf{k}) c^\dagger_\sigma(\mathbf{k}) c_\sigma(\mathbf{k})\,, \tag{1}$$

where $c^\dagger_\sigma(\mathbf{k})$ and $c_\sigma(\mathbf{k})$ are the usual fermionic creation and annihilation operators. For a charge nematic, the form factor $d(\mathbf{k})$ has a $\mathbf{k}$-dependence with d-wave symmetry, such as $d(\mathbf{k}) = \cos k_x - \cos k_y$. Decoupling the fermionic interaction by introducing an order parameter field ϕ via a Hubbard-Stratonovich transformation, and integrating out the fermionic variables, one obtains an effective action $S[\phi]$. The effective N-point interactions of the order parameter field are given by symmetrized N-point loops

$$\Gamma_N(q_1, \ldots, q_N) = \frac{1}{N!} \sum_P \Pi_N(q_{P1}, \ldots, q_{PN})\,, \tag{2}$$

where the sum collects all permutations P of $1, \ldots, N$, and

$$\Pi_N(q_1, \ldots, q_N) = \int_k \prod_{j=1}^N \left[d(\mathbf{k} - \mathbf{p}_j - \mathbf{q}_j/2)\, G_0(k - p_j) \right]\,. \tag{3}$$

We use 3-vectors collecting imaginary frequency and two-dimensional momentum variables, for example $k = (k_0, \mathbf{k})$, and $\int_k$ as a short-hand notation for $\int \frac{dk_0}{2\pi} \frac{d^2k}{(2\pi)^2}$. The variables p_j and q_j are related by $q_j = p_{j+1} - p_j$ for $j = 1, \ldots, N-1$, and $q_N = p_1 - p_N$. Note that $q_1 + \cdots + q_N = 0$ due to energy and momentum conservation. The bare propagator has the form $G_0(k) = [ik_0 - \epsilon(\mathbf{k}) + \mu]^{-1}$, where $\epsilon(\mathbf{k})$ is the dispersion relation of the non-interacting particles. $\Pi_N(q_1, \ldots, q_N)$ can be represented graphically as a fermion loop with N lines corresponding to G_0 and N vertices with form factors $d(\mathbf{k})$, as shown in Fig. 1.

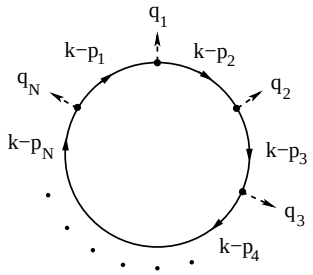

Figure 1: Graphical representation of Π_N with momentum variables as in Eq. (3).

The bare fluctuation propagator has the form

$$D_0(q) = \frac{1}{\chi \mathbf{q}^2 + \gamma \frac{|q_0|}{|\mathbf{q}|}}\,, \tag{4}$$

where χ and γ are positive constants. $D_0(q)$ diverges in the limit $\mathbf{q} \to 0$ and $q_0/|\mathbf{q}| \to 0$. The two terms in the denominator of $D_0(q)$ vanish at the same pace for $\mathbf{q} \to 0$ if $q_0 \propto |\mathbf{q}|^3$. To assess the size of the interaction terms in $S[\phi]$ one thus has to study the N-point loops in a low-energy limit with $q_{j0} \propto |\mathbf{q}_j|^3$. Naively one may expect that this corresponds to the *static* limit, where $q_{j0} \to 0$ before $\mathbf{q}_j \to 0$:

$$\lim_{\mathbf{q}_j \to 0} \lim_{q_{j0} \to 0} \Pi_N(q_1, \ldots, q_N) = \frac{(-1)^{N-1}}{(N-1)!} \frac{\partial^{N-2}}{\partial \mu^{N-2}} \int \frac{d^2k}{(2\pi)^2} [d(\mathbf{k})]^N \, \delta[\epsilon(\mathbf{k}) - \mu]\,. \tag{5}$$

Except for special cases where the chemical potential lies at a van Hove singularity, this expression is ***finite***. Note that the right hand side of Eq. (5) is independent of $q_1, \ldots, q_N$ and hence already symmetrized. Approximating the bosonic N-point interactions by finite local interactions thus seems adequate. Standard power counting then implies that all interactions with $N \geq 4$ are increasingly (with higher N) irrelevant. Hence, the Hertz-Millis truncation seems justified. The static limit of the 3-point loop and all other N-point loops with odd N as given by Eq. (5) vanishes in the case of a charge nematic, due to the antisymmetry of $d(\mathbf{k})$ under $\pi/2$ rotations of $\mathbf{k}$. One arrives at a similar conclusion for the gauge field problem.

However, there is a flaw in the above argument. Eq. (5) has been derived by setting $q_{j0} = 0$ before the momenta $\mathbf{q}_j$ tend to zero. It is not guaranteed that this captures the low-energy limit $\mathbf{q}_j \to 0$ and $q_{j0}/|\mathbf{q}_j| \to 0$ in general. Indeed, a simple estimate indicates that the N-point loop is of order $q_{j0}/|\mathbf{q}_j|^{N-1}$ for small non-collinear momenta $\mathbf{q}_j$ and small $q_{j0}/|\mathbf{q}_j|$ [1]. Although this behavior is increasingly singular for larger N, the corresponding order parameter interactions remain irrelevant, since the singularity is not strong enough. However, an even stronger singularity is obtained in a special low-energy limit in which the momenta $\mathbf{q}_1, \ldots, \mathbf{q}_N$ become *collinear*. The crucial role of coupled fluctuations with collinear momenta was highlighted by Metlitski and Sachdev [1]. In perturbative one-loop calculations of the fermionic self-energy $\Sigma(\mathbf{k}_F, \omega)$ at a certain point $\mathbf{k}_F$ on the Fermi surface, it was found already some time ago that the dominant contributions involve only fermionic states in the momentum region near $\mathbf{k}_F$ and $-\mathbf{k}_F$, with momentum transfers $\mathbf{q}$ almost *tangential* to the Fermi surface in those points. This remains true for higher order contributions, so that all fermionic momenta are close to $\mathbf{k}_F$ and $-\mathbf{k}_F$ and momentum transfers are almost tangential to the Fermi surface in these points, which implies that they are mutually almost collinear.

Choosing a coordinate system in momentum space in such a way that the normal vector to the Fermi surface at $\mathbf{k}_F$ points in x-direction, the proper scaling limit describing the low-energy behavior is given by $k_0 \mapsto \lambda^3 k_0$, $k_x \mapsto \lambda^2 k_x$, and $k_y \mapsto \lambda k_y$ with $\lambda \to 0$, where (k_x, k_y) is measured relative to $\mathbf{k}_F$. For the momentum and energy transfers q_j this implies

$$q_{j0} \mapsto \lambda^3 q_{j0}\,, \quad q_{jx} \mapsto \lambda^2 q_{jx}\,, \quad q_{jy} \mapsto \lambda q_{jy} \quad \text{with} \quad \lambda \to 0\,. \tag{6}$$

In this *collinear low-energy limit* the momentum transfers $\mathbf{q}_j$ become increasingly collinear (pointing in y-direction). We have computed the exact scaling behavior of the effective N-point interactions in that limit [3]. The calculation is rather complicated, but the result is very simple:

$$\Gamma_N \propto \lambda^{6-2N} \quad \text{for all } N \geq 3\,. \tag{7}$$

The degree of divergence therefore increases rapidly with N. It is not reduced by any cancellations. By standard power-counting one then finds that all N-point interactions are marginal, that is, they cannot be neglected in the low-energy limit. Truncating the effective action $S[\phi]$ at quartic order in ϕ as in the Hertz-Millis approach is therefore not justified. We have also derived explicit expressions for the full momentum and frequency dependence of $\Gamma_N(q_1, \ldots, q_N)$ in the collinear low-energy limit [3].

References:

[1] *Metlitski, M.A. and S. Sachdev.* Physical Review B **82**, 075127 (2010).

[2] *Neumayr, A. and W. Metzner.* Physical Review B **58**, 15449-15459 (1998).

[3] *Thier, S.C. and W. Metzner.* Physical Review B **84**, 155133 (2011).